International Journal of Modern Physics D
Vol. 24, No. 5 (2015) 1530014 (53 pages)

DOI: 10.1142/S0218271815300141

Unification of inflation and dark energy *à la* quintessential inflation*

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Received 31 October 2014
Revised 15 January 2015
Accepted 16 January 2015
Published 3 March 2015

This pedagogical review is devoted to quintessential inflation, which refers to unification of inflation and dark energy using a single scalar field. We present a brief but concise description of the concepts needed to join the two ends, which include discussion on scalar field dynamic, conformal coupling, instant preheating and relic gravitational waves. Models of quintessential inflation broadly fall into two classes, depending upon the early and late time behavior of the field potential. In the first type we include models in which the field potential is steep for most of the history of the universe but turn

*IJMPD invited review, dedicated to 76th birthday of J.V. Narlikar, based upon the lecture delivered by M. Sami at IUCAA in July 2014.

shallow at late times, whereas in the second type the potential is shallow at early times followed by a steep behavior thereafter. In models of the first category inflation can be realized by invoking high-energy brane-induced damping, which is needed to facilitate slow roll along a steep potential. In models of second type one may invoke a nonminimal coupling of the scalar field with massive neutrino matter, which might induce a minimum in the potential at late times as neutrinos turn nonrelativistic. In this category we review a class of models with noncanonical kinetic term in the Lagrangian, which can comply with recent B mode polarization measurements. The scenario under consideration is distinguished by the presence of a kinetic phase, which precedes the radiative regime, giving rise to blue spectrum of gravity waves generated during inflation. We highlight the generic features of quintessential inflation and also discuss on issues related to Lyth bound.

Keywords: Variable gravity; quintessential inflation; gravitational waves.

PACS Number(s): 98.80.−k, 98.80.Cq, 95.36.+x, 04.50.Kd

1. Introduction

The list of great successes of the standard model of the universe, dubbed hot big bang, includes its predictions about the universe expansion,[1] the existence of microwave background[2] and the synthesis of light elements in the early universe.[3–13] In the model, there is a profound mechanism of clustering, via gravitational instability, provided primordial density perturbations are assumed. The generation of tiny fluctuations observed by COBE in 1992,[14] required for structure formation, are beyond the scope of hot big bang. Additionally, the standard model also suffers from inherent logical inconsistencies such as the flatness problem, the horizon problem and others, which imply the incompleteness of the scenario. Inflation[15–19] is a beautiful paradigm which not only addresses the said shortcomings but also provides us a quantum mechanical generation mechanism for primordial fluctuations— scalar (density) perturbations and tensor perturbations or primordial gravitational waves.[20–28]

Inflation predicts a nearly flat spectrum of density perturbations, whose amplitude needs to be fixed using COBE normalization.[29,30] Perhaps the clearest prediction of inflation is related to the generation of gravitational waves at its end. The relic gravitational waves[31–57] can give rise to B mode polarization of CMB, which depends upon the tensor-to-scalar ratio of perturbations r.[58–60] The recent BICEP2 measurements reveal that $r \simeq 0.2$,[61] thereby the amplitude of gravitational waves is sizeable such that the scale of inflation is around the GUT scale. The large value of r, in the framework of single-field inflation is directly related to the range of inflation, giving rise to super Planckian excursion of the inflaton field,[22,62–66] which throws a challenge to effective field theoretic description of inflation. Even if the BICEP2 results are not confirmed, it looks quite likely that $r = 0$ would stand ruled out, thereby strengthening the belief that inflation is a viable early time completion of the standard model of universe.

There is one more shortcoming the hot big bang is plagued with, namely the age crisis, which is related to the late-time evolution of the universe.[67–69] The only way to circumvent the problem in the standard lore is to add a repulsive effect, triggered by a positive cosmological constant,[70] or by a slowly rolling scalar field with mass of the order of H_0 called quintessence.[71–81] Thus, the resolution of the age-related inconsistency asks for late time cosmic acceleration[82–92] discovered in 1998 by supernovae Ia observations[93–95] and supported indirectly thereafter by other probes.[96–107]

It is amazing that both the early and late time completions of the standard model of the universe require accelerated expansion. Often, these two phases of acceleration are treated separately. It is tempting to think that there is a unique cause responsible for both the phases, or the late time cosmic acceleration is nothing but the reincarnation of inflation, and such a paradigm is known as *quintessential inflation*[48–50,108,109] (see also Refs. 110 and 152). In simple cases inflation is driven by a scalar field, which soon after the end of inflation enters into an oscillatory phase and fastly decays in particle species giving rise to reheating/preheating of the universe.[153–160] However, if the single scalar field is to unify both early and late time cosmic acceleration, it should survive till the late times, thus the conventional reheating would fail in this case. The second obstacle to unification is related to a very accurate description of the thermal history of the universe by the big bang model. Indeed, invoking a new degree of freedom over and above the standard model of particle physics should be sufficiently suppressed to be consistent with nucleleosynthesis constraint. Clearly the scalar field should evolve in a specific manner to accomplish the task of joining the two ends: It should evolve very slowly at early times followed by fast roll after inflation such that it goes into hiding for most of the history of universe. It should reappear only around the present epoch to account for the late time acceleration. It is desirable that late time evolution should have no memory about the initial conditions, which requires a specific scalar field dynamics. The desired field evolution can be guaranteed by a field potential which is effectively shallow at early times, followed by steep behavior of approximately exponential type giving rise to scaling regime such that the field mimics the background. The late time features in the potential should then trigger the exit from the scaling regime.

Broadly, there are two types of models in which unification of inflation and quintessence can be achieved. First, models which use field potentials that are steep except at late time where they turn (effectively) shallow. In this case extra damping is required to facilitate the slow roll in the early phase. In the Randall–Sundrum brane worlds,[161–163] the high energy corrections to Einstein equations can provide the required damping facilitating slow roll along a steep potential, such that the high energy effect disappears as the field rolls down its potential allowing for a graceful exit from inflation.[48–50,109–111,164–167] The post inflationary dynamics in this case would be as desired, though the tensor-to-scalar ratio is somewhat larger than its recently measured values.

The second option is provided by models based upon field potentials which are shallow in the early phase followed by scaling behavior thereafter. It is easier to cast such a class of models using noncanonical kinetic terms in the Lagrangian. The exit from scaling solution at late times can be triggered in this case by the presence of nonminimal coupling to massive neutrino matter.[146,147,168] For neutrinos with masses around 1 eV, the coupling to field builds up around the present epoch, leading to minimum in the potential which is otherwise of run-away type. If field rolls slowly around the minimum, we may obtain a desired late time behavior.[146,168]

Quintessential inflation possesses certain general features: (i) Standard reheating mechanism is not applicable in this case. (ii) Post inflationary dynamics is governed by the kinetic regime. The first aspect poses a problem, whereas the second one provides an excellent perspective which could allow to falsify the scenario of quintessential inflation irrespectively of the underlying model. However, both problems and prospects are intrinsically related to each other. One of the known nonconventional reheating mechanisms could be achieved via gravitational particle production. Nevertheless, it is an inefficient process leading to long kinetic regime before the commencement of radiation domination. And here comes the punch line since the evolution of gravitational waves generated during inflation crucially depends on the post-evolutionary equation of state.[35,37,38,45] During radiation and matter dominated epochs the relic gravitational waves track the background, but during the kinetic regime the ratio of energy density in gravitational waves to the background energy density enhances and might conflict with the nucleosynthesis constraint at the commencement of radiative regime, depending upon the duration of the kinetic regime. This is what happens in the case of gravitational particle production. The instant preheating mechanism[169–171] can circumvent the problem. Let us emphasize that one of the generic prediction of quintessential inflation, irrespectively of the underlying model, is the blue spectrum of relic gravitational waves produced during the transition from inflation to kinetic regime, which could be tested by observations like Advanced LIGO and LISA.

The present review is dedicated to quintessential inflation and aims for both the young researchers and experts. All the essential ingredients required to implement the underlying idea are described; the exposition is coherent and pedagogical. In Sec. 2, we give the building blocks of quintessential inflation, and we present a brief account of scaling/tracker solutions and dynamics of nonminimally coupled scalar field. Moreover, we include a discussion on the difficulties associated with the fundamental scalar field *à la* naturalness. As a prerequisite to quintessential inflation we herewith include the essentials of relic gravitational waves and instant preheating. In Sec. 3, we review the steep braneworld inflation and its unification with dark energy. The last subsection of Sec. 3 is devoted to quintessential inflation described by Lagrangians with a noncanonical form and nonminimally coupling to massive neutrino matter.

Last but not the least, a brief guideline for reading the review and its follow up is in order. Readers not acquitted with the theme are advised to read through subsections of Sec. 2. Results in subsection on scalar field dynamics can easily be worked out. Concerning the subsection on conformal transformation, in case the reader is interested in details, we recommend to read it with the help of Refs. 172 and 173. Subsection on relic gravitational waves is a bit technical. In the first reading one might go through it leaving the details aside. Readers interested in more details are referred to Refs. 34 and 35, as well as to later works.[37,38,45,48,174] Experts may directly begin with Sec. 3. While reaching Sec. 3 we recommend in the first reading to begin directly from the Einstein frame action (118). Throughout the manuscript we use the metric signature $(-,+,+,+)$, and conventions $R = g^{\alpha\beta}R_{\alpha\beta}; R_{\mu\nu}R^{\alpha}_{\mu\alpha\nu}; R^{\mu}_{\nu\alpha\beta} = \partial_\alpha\Gamma^{\mu}_{\nu\beta} + \cdots$. Finally, we use the system of units $\hbar = c = 1$ and the notation $8\pi G = M_{\rm Pl}^{-2}$.

2. Building Blocks and Ingredients of Quintessential Inflation

As mentioned in the Introduction, one needs specific features of scalar field dynamics such that the traditional big bang evolution is sandwiched between two phases of accelerated expansion. It is desirable that the dynamics be insensitive to a broad choice of initial conditions. In what follows we shall describe scaling solutions and late time exit from them *à la* tracking behavior.[175]

2.1. *Scalar field dynamics, attractors and late time acceleration*

For our purpose we need a slowly-rolling field in the beginning followed by fast roll thereafter, till late times where slow roll again needs to be commenced. In the presence of background (matter/radiation) we aim to find solutions of interest to quintessential inflation. Let us first consider a minimally coupled scalar field, with action

$$\mathcal{S} = -\int \left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]\sqrt{-g}\,\mathrm{d}^4x. \tag{1}$$

The energy–momentum tensor corresponding to action (1) is given by

$$T_{\mu\nu} \equiv -2\frac{1}{\sqrt{-g}}\frac{\delta\mathcal{S}}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right]. \tag{2}$$

Specializing to spatially flat homogeneous and isotropic background,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{3}$$

one obtains the expressions of pressure and energy density of the scalar-field system as

$$\rho_\phi \equiv T^0_0 = \frac{\dot\phi^2}{2} + V(\phi); \quad p_\phi \equiv T^1_1 = \frac{\dot\phi^2}{2} - V(\phi). \tag{4}$$

The Euler Lagrangian equation for the action (1) in the FRW background ($\sqrt{-g} = a^3$), acquires the simple form

$$\ddot\phi + 3H\dot\phi + V'(\phi) = 0 \Rightarrow \dot\rho_\phi + 3H\rho_\phi(1 + w_\phi) = 0, \tag{5}$$

where a prime denotes the derivative with respect to the field, dots denote derivatives with respect to the cosmic time, H is the Hubble parameter and $w_\phi = p_\phi/\rho_\phi$ is the equation of state parameter for the field. The Friedmann equation is written as

$$3H^2 M_{\rm Pl}^2 = \rho_\phi, \tag{6}$$

where we have ignored other components of energy density present in the universe.

The equation of motion (5) formally integrates to

$$\rho_\phi = \rho_{\phi 0} e^{-3\int (1+w_\phi)\frac{da}{a}} \to \rho_\phi \sim a^{-n}, \quad 0 \le n \le 6, \tag{7}$$

where $n = 0$ corresponds to $w_\phi = -1$ (cosmological constant), whereas the other limiting case relates to $w_\phi = 1$ (stiff matter) which can be realized by slowly (fast) rolling scalar field along a flat (steep) potential.

As mentioned in the Introduction, we are interested in specific solutions of scalar field dynamics, in presence of the background energy density (radiation/matter) $\rho_{\rm b}$, in which case the Friedmann equation becomes

$$3H^2 M_{\rm Pl}^2 = \rho_\phi + \rho_{\rm b}. \tag{8}$$

In order to exhibit the interesting features of the dynamics we cast the evolution equations in autonomous form, by invoking the dimensionless variables[78,82,176–183]

$$x = \frac{\dot\phi}{\sqrt{6} M_{\rm Pl} H}, \quad y = \frac{\sqrt{V}}{\sqrt{3} M_{\rm Pl} H}, \quad \lambda = M_{\rm Pl}\frac{V'}{V}, \quad \Gamma = \frac{VV''}{V'^2}. \tag{9}$$

The evolution equations obtain the form

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3}{2}x[(1-w_{\rm b})x^2 + (1+w_{\rm b})(1-y^2)], \tag{10}$$

$$\frac{dy}{dN} = \frac{\sqrt{6}}{2}\lambda xy + \frac{3}{2}y[(1-w_{\rm b})x^2 + (1+w_{\rm b})(1-y^2)], \tag{11}$$

$$\frac{d\lambda}{dN} = -\sqrt{6}\lambda^2(\Gamma - 1)x, \tag{12}$$

where $N = \ln(a)$, while the Friedmann equation yields the constraint equation

$$x^2 + y^2 + \frac{\rho_{\rm b}}{3M_{\rm Pl}^2 H^2} = 1. \tag{13}$$

The equation of state and the dimensionless density parameter are conveniently expressed through x and y as

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{x^2 - y^2}{x^2 + y^2}; \quad \Omega_\phi \equiv \frac{\rho_\phi}{3M_{\rm Pl}^2 H^2} = x^2 + y^2. \tag{14}$$

Let us first consider a particularly important case when $\Gamma = 1$, implying a constant slope of potential $\lambda =$ const. that corresponds to an exponential potential, namely

$$V(\phi) = V_0 e^{-\frac{\lambda\phi}{M_{\rm Pl}}}, \tag{15}$$

in which case the last equation (12) decouples from the system of autonomous equations. We then extract the fixed points by setting $dx/dN = 0$ and $dy/dN = 0$.

The fixed points which are relevant are those around which the perturbations die out exponentially, namely the stable points. In case of an exponential potential, we have two stable fixed points

$$\textbf{1}.\quad x = \frac{\lambda}{\sqrt{6}};\quad y = \sqrt{1 - \frac{\lambda^2}{6}};\quad w_\phi = \frac{\lambda^2}{3} - 1;\quad \Omega_\phi = 1,\quad \lambda^2 < 3(1 + w_{\rm b}), \tag{16}$$

$$\textbf{2}.\quad x = \left(\frac{3}{2}\right)^{1/2} \frac{1 + w_{\rm b}}{\lambda};\quad y = \left(\frac{3(1 - w_{\rm b}^2)}{2\lambda^2}\right)^{1/2};\quad w_\phi = w_{\rm b};$$

$$\Omega_\phi = \frac{3(1 + w_{\rm b})}{\lambda^2};\quad \lambda^2 > 3(1 + w_{\rm b}). \tag{17}$$

The first fixed point corresponds to field-dominated solution which is stable provided that $\lambda^2 < 3(1 + w_{\rm b})$ and gives rise to acceleration in case of $\lambda < \sqrt{2}$, which is well known from slow roll conditions. The second fixed point is very interesting and exists for a steep potential. This solution dubbed *scaling solution*[78,176,177,184] mimics the background such that $\rho_\phi/\rho_{\rm b} = \text{const}$ (see Fig. 1(a)). A scaling solution is desired for most of the history of the universe.

Let us note that fixed points (1) and (2) are mutually excluding. In a realistic scenario, in certain sense, we need both of them together. In that case we need a feature in the potential that could allow to exit from the scaling solution and get into the field-dominated solution described by fixed point (2). Clearly, we need to go beyond exponential potential, in a way that the potential mimics a steep-exponential-like behavior for most of the time and turns shallow at late times to mimic the first fixed point.

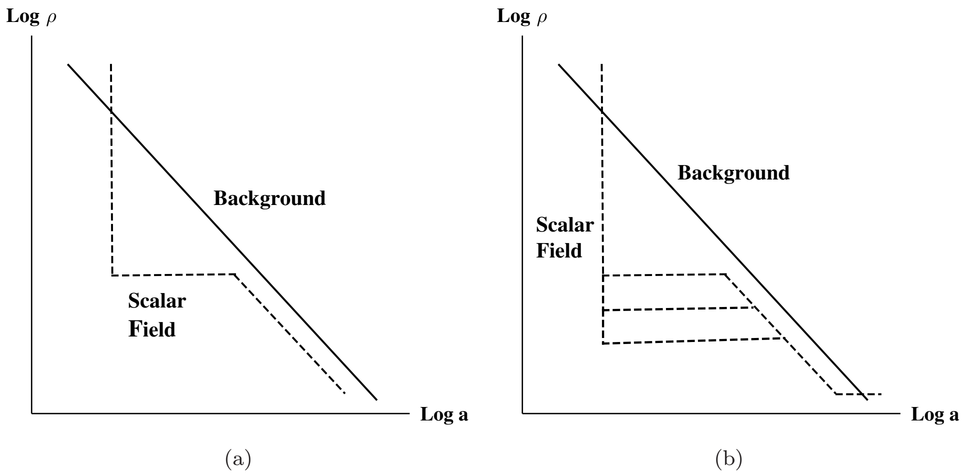


Fig. 1. Schematic diagram of scaling (a) and tracker (b) behavior. Different dashed lines in the Fig. 1(b) correspond to different initial conditions for the field. This signifies that field joins the tracker sooner or later depending upon the initial conditions.

Let us now move away from $\Gamma = 1$ or the exponential potential. In case $\Gamma > 1$ the slope decreases from higher values to zero, giving rise to accelerated expansion at late times. The condition $\Gamma > 1$ is regarded as the tracking condition under which the field energy density eventually catches up with the background. When $\Gamma < 1$ the slope increases, and since the potential is steep in this case the energy density of the scalar field becomes negligible compared to that of the background energy density. This case is not interesting in view of accelerated expansion at late times. In order to construct viable quintessence models, we require that the potential should satisfy the tracking condition. For instance, $\Gamma = (n + 1) = n > 1$ in case of the inverse power-law potentials $V(\phi) \sim \phi^{-n}$ with $n > 0$. This implies that the tracking occurs for this potential. In this case the field rolls from small values toward infinity and thereby the potential is steep at early epochs and turn shallow at late times. Since inverse power-law potentials are intimately related to exponential behavior, the field approximately mimics the background and at late times it exits to acceleration as potential turns shallow. This is exactly the desired behavior we are looking for. Such a behavior can also be realized in case of double exponential and *cosh* potentials.

Let us understand the tracking behavior through Figs. 1(a) and 1(b). Initially, the scalar-field energy density is much larger than the background energy density and the potential is steep. As a result, the field runs down its potential fast, making the potential energy irrelevant, and undershoots the background. The Hubble damping in (5) becomes large as $\rho_{\mathrm{b}} \gg \rho_\phi$ and thus the field freezes on the potential mimicking the cosmological-constant-like behavior in both Figs. 1(a) and 1(b). At the same time the background energy density redshifts and the field waits till it becomes comparable to its energy density, and when this happens the field resumes its motion. Supposing that undershoot is such that the field is still in the steep region of the potential, in this case it enters the scaling regime and tracks the background before reaching the shallow region of the field potential. As it reaches this region, which can be made to happen around the present epoch, its motion slows down and the field energy density overtakes the background giving rise to late-time cosmic acceleration [Fig. 1(b)]. Once this behavior is set correctly around the present epoch by making the appropriate choice of model parameters, the evolution is insensitive to initial conditions in a wide range of them. In case of nontracker dark energy models, when the field resumes evolution starting from the freezing regime, it just takes over the background without following it. These models are plagued with the same level of fine-tuning problem as ΛCDM itself. The tracker models might look attractive at the onset. In what follows, we shall try to convince the reader that the problem is deep and cannot be addressed so simply.

2.2. *Scalar field and naturalness*

In this subsection we shall briefly demonstrate that models containing a fundamental scalar field similar to standard model of particle physics are faced with the problem of naturalness. It is expected that in a healthy field theoretic set up, physics at lower mass scales gets decoupled from higher energy scales *à la naturalness*. The criteria of naturalness, formulated by t'Hooft, state that a parameter α in a field theory is natural if by switching it off in the Lagrangian leads to enhancement of symmetry, which is respected at the quantum level too.[185] In such theories, the quantum correction should be in the form, $\delta\alpha \sim \alpha^n (n > 0)$. Theories such as quantum electrodynamics and quantum chromodynamics satisfy the criteria of naturalness. Quantum electrodynamics, in particular, is a successful description of atomic physics via interaction of electrons and photons without any knowledge of higher mass scales associated with heavier leptons and quarks.

A field theory that includes a fundamental scalar violates this important property. In these theories, the quantum correction to the mass of the scalar is proportional to the highest mass scale in the theory thereby lower scales get dragged towards the highest scale. In this case even if the symmetry is enhanced at classical level, the same is not respected at quantum level.[186–190,a] The latter is closely related to the cosmological constant problem. In presence of a cosmological constant alone, we obtain the de Sitter space as a solution to Einstein equations. If we switch it off, the flat space time becomes a solution, which is characterized by the Poincare symmetry with 10 generators ($SO(3,1)$ − three rotations, three Lorentz boosts and 4 translations). In case of de Sitter space, the symmetry group is $SO(4,1)$ which has 6 rotations and 4 Lorentz boosts. Thus, symmetry is not enhanced in this case. As for quantum corrections, any massive field placed in vacuum contributes to vacuum energy whose mass scale is proportional to the highest fundamental mass scale. Hence, the cosmological constant is not a natural parameter of Einstein theory.

Let us first consider the cosmological constant problem. Sakharov, in 1968,[193] first pointed out that the vacuum expectation value of energy–momentum tensor of a field placed, by virtue of relativistic covariance, has the following form:

$$\langle 0|T_{\mu\nu}|0\rangle = -\rho_v g_{\mu\nu}, \tag{18}$$

where ρ_v is a generic constant due to conservation of energy-momentum tensor. Assuming the perfect fluid form for $T_{\mu\nu}$,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}; \quad u^\mu = (1,0,0,0), \tag{19}$$

where

$$\rho = u^\mu u^\nu T_{\mu\nu}; \quad p = \frac{1}{3}\mathcal{P}^{\mu\nu}T_{\mu\nu}; \quad \mathcal{P}^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu, \tag{20}$$

then relativistic covariance [see Eqs. (18) and (19)], demands that $\rho_v = -p_v$.

[a]We thank R. Kaul for many useful discussions on this theme.

This quantum correction (18) should be added to the right hand side of Einstein equations (see, Ref. 191 for an alternative point of view)

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda_{\rm B} = M_{\rm Pl}^{-2}(T^{\rm m}_{\mu\nu} + \langle 0|T_{\mu\nu}|0\rangle), \tag{21}$$

where $\Lambda_{\rm B}$ is the bare value of the cosmological constant. Note that observations measure the effective value

$$\Lambda_{\rm eff} = \Lambda_{\rm B} + M_{\rm Pl}^{-2}\rho_v. \tag{22}$$

The quantity ρ_v can be estimated by imagining the field as a collection of harmonic oscillators and by summing up their zero point energy as[192]

$$\rho_v = \frac{1}{2}\frac{1}{(2\pi)^3}\int {\rm d}^3\mathbf{k}\,\omega(k), \tag{23}$$

$$p_v = \frac{1}{6}\frac{1}{(2\pi)^3}\int {\rm d}^3\mathbf{k}\frac{k^2}{\omega(k)}, \tag{24}$$

$$\omega(k) = \sqrt{k^2+m^2}, \tag{25}$$

where m is the mass of the field and $k^\mu = (k^0, \mathbf{k})$ with $\mathbf{k} = k$. We have dropped the spin factor which does not change the order of magnitude of vacuum energy. Using Eqs. (23) and(24) we can write

$$\langle T\rangle = -\rho_v + 3p_v = -\frac{1}{2}\frac{1}{(2\pi)^3}\int {\rm d}^3\mathbf{k}\frac{m^2}{\omega(k)}. \tag{26}$$

Next, let us confirm that ρ_v corresponds to vacuum bubble diagram. The vacuum bubble is described by the Feynman propagator $D_{\rm F}(0)$[192]

$$D_{\rm F}(0) = \frac{i}{(2\pi)^4}\int\frac{{\rm d}^4k}{k^2+m^2} = \frac{i}{(2\pi)^4}\int\frac{{\rm d}^0k{\rm d}^3\mathbf{k}}{-k_0^2+\omega^2}. \tag{27}$$

Using then the identity

$$\int\frac{{\rm d}^0k}{-k_0^2+\omega^2} = i\frac{\pi}{\omega}, \tag{28}$$

we have

$$D_{\rm F}(0) = -\frac{1}{2}\frac{1}{(2\pi)^3}\int\frac{{\rm d}^3\mathbf{k}}{\omega}. \tag{29}$$

Comparing Eqs. (26) and (29) we get

$$\langle T\rangle = m^2 D_{\rm F}(0). \tag{30}$$

Remembering that $p_v = -\rho_v$ and using Eq. (30), we finally arrive at[192]

$$\rho_v = -\frac{m^2}{4}D_{\rm F}(0). \tag{31}$$

Hence computation of vacuum energy is directly related to the vacuum bubble with massive field circulating in it. We should then sum up the contribution from all the massive fields circulating in the bubble. It is clear that the highest mass scale gives the leading contribution. ρ_v is formally quadratic divergent and we can estimate it by using the dimensional regularization. Subtracting out the divergent part we acquire

$$\rho_v \simeq \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right), \tag{32}$$

with μ an arbitrary scale to be fixed from observations which is however not very important. The crucial information is contained in the logarithmic pre-factor. If we believe that there is no physics beyond the standard model of particle physics, we might identify m with the mass of the top quark to obtain the leading contribution. It is important to note that even if we turn the cosmological constant to zero at the classical level, it will be generated by quantum corrections which is generically a large value. Hence, the cosmological constant is not a natural parameter of Einstein theory. Finally, we mention here that higher loop diagrams will not add anything new, they will simply renormalize m.

Before we proceed ahead, let us comment on the Lorentz invariant character of ρ_v. Which is obvious from (18) and (31). However, since (23) is an ultraviolet divergent quantity, ρ_v might become frame dependent in case the cut off does not respect Lorentz invariance. It is therefore necessary that one invokes a suitable scheme such as dimensional regularization, consistent with Lorentz symmetry, for the computation of vacuum energy.

Let us now turn to field theory where a scalar field couples to a massive fermion:

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2 + \bar{\Psi}(i\gamma^\mu\partial_\mu - m_\Psi)\Psi + g\phi\bar{\Psi}\Psi, \tag{33}$$

with $m_\Psi \gg m$. If we now compute the one-loop correction to m, we encounter quadratic divergence. Using then dimensional regularization and carrying out the substraction, we find

$$\delta m^2 \sim g^2 \int \mathrm{d}^4k \frac{k^2 - m_\Psi^2}{(k^2 + m_\Psi^2)^2} \sim g^2 m_\Psi^2 \ln\left(\frac{m_\Psi^2}{\mu^2}\right). \tag{34}$$

The quantum correction is proportional to the heaviest mass scale and does not disappear in the limit $m \to 0$, therefore the mass of the scalar is not protected under radiative corrections and it gets dragged towards the heaver mass scale of fermions. This is a similar situation with the one we encountered in the cosmological constant case. Hence, naturalness is lost in a model that contains a fundamental scalar.

The situation is quite different in quantum electro dynamics (QED), where the action reads

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - m_{\mathrm{e}}]\Psi. \tag{35}$$

In this case $m_{\mathrm{e}} \to 0$ enhances the symmetry of the Lagrangian, namely the chiral symmetry appears. The one-loop correction to the electron mass is logarithmically divergent, and using a similar procedure we find

$$(\delta m_{\rm e})_{\rm one\ loop} \sim {\rm e}^2 m_{\rm e} \ln\left(\frac{\mu}{m_{\rm e}}\right), \tag{36}$$

with a remarkable property that the correction disappears in the limit $m_{\rm e} \to 0$. If we invoke heavier fermions, their contribution is suppressed by inverse powers of the heavier scales, rendering the theory natural. It is this property that allows for the decoupling of heavy mass scales from low-mass-scale phenomena in QED, and thus atomic physics can safely be done without the knowledge of heavy flavors. In case of the standard model, the Higgs particle mass, the mass of gauge bosons and fermion masses are all proportional to the vacuum expectation value of the Higgs field. Turning the vacuum expectation value to zero at classical level, enhances the symmetry. However, at quantum level, the vacuum expectation value gets generated by quantum correction, which renders the theory unnatural. This implies that there is physics beyond the standard model. One way to UV completion is to invoke supersymmetry which can restore the naturalness of the theory.

In the context of cosmology, since inflation occurs at high energy scales, inflation can be protected by supersymmetry. Recent observations have ultimately confirmed the late-time cosmic acceleration, which could be fueled by cosmological constant or equivalently by a slowly rolling scalar field of mass of the order of $H_0 \sim 10^{-33}$ eV. At such low energies, there is no known symmetry that could protect the cosmological constant or quintessence. Do we require a completion of the theory at this end? There is no known way of restoring the naturalness of the theory at low energies. Closing our eyes on this problem, we shall proceed to work with models that essentially contain a fundamental scalar field, for instance the modified theories of gravity.

2.3. *Conformal transformation and nonminimally coupled scalar field system*

Modified theories of gravity have been investigated recently in the contexts of inflation as well as late-time cosmic acceleration. An important class of modified theories is described by scalar-tensor theories, which apart from the spin-2 object also contain a scalar degree of freedom. One of such schemes was first proposed by Brans and Dicke. In general, these theories can be described either in Jordan frame or in Einstein frame. In the Jordan frame, the particle masses are generic constants and the matter energy–momentum tensor is conserved on its own, but the scalar degree of freedom is kinetically mixed with the metric. On the other hand, in Einstein frame the Lagrangian is diagonalized but the scalar field is directly coupled to matter, thereby the matter energy momentum tensor is not conserved. The field equation of motion also gets modified such that the total energy–momentum tensor is still conserved. The two frames are connected to each other by virtue of a conformal transformation. One important consequence of nonconservation of matter energy–momentum tensor manifests in the transformation of particle masses under conformal transformation. Since conformal transformation is not a symmetry of the Lagrangian in general, the question about the equivalence of the two frames naturally arose in the literature. Some authors claimed that the physical frame is the Jordan one whereas others considered the Einstein frame to be the physical one. The confusion existed in the literature till very recently before the issue was settled in Refs. 194 and 195. One can show that not only mathematically, but also physically both frames are equivalent: *Physical quantities do change under conformal transformation but the relationship between physical observables remains the same in both frames.*

Let us consider the following Brans–Dicke action

$$\mathcal{S}_{\rm BD} = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\,\frac{1}{2}\left[M_{\rm Pl}^2\varphi\tilde{R} - \frac{M_{\rm Pl}^2\omega_{\rm BD}(\varphi)}{\varphi}\left(\tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\right) - 2U(\varphi)\right] + \int \mathrm{d}^4x\sqrt{-\tilde{g}}\mathcal{L}_m(\psi,\tilde{g}_{\mu\nu}), \tag{37}$$

where $\omega_{\rm BD}(\varphi)$ is known as Brans–Dicke parameter. It should be noted that the field does not couple directly to matter (it does not appear in the matter action). However, the scalar degree of freedom does mix with the curvature or the spin-2 object in the metric $\tilde{g}_{\mu\nu}$, dubbed Jordan metric, and the action (37) is then said to be in the Jordan frame. The equations of motion for the gravitational sector can be obtained by varying the action (37) with respect to $\tilde{g}_{\mu\nu}$ in the Jordan frame, namely

$$\varphi\tilde{G}_{\mu\nu} = M_{\rm Pl}^{-2}\tilde{T}_{\mu\nu} + \frac{\omega_{\rm BD}(\varphi)}{\varphi}\left[\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\tilde{g}_{\mu\nu}(\tilde{\nabla}\varphi)^2\right] + \tilde{\nabla}_\mu\tilde{\nabla}_\nu\varphi - \tilde{g}_{\mu\nu}\tilde{\Box}\varphi - M_{\rm Pl}^{-2}\tilde{g}_{\mu\nu}U, \tag{38}$$

where $\tilde{\nabla}$ and $\tilde{\Box}$ are the covariant derivative and the Laplacian operator respectively defined using the metric $\tilde{g}_{\mu\nu}$. Equation (38) is quite complicated due to the mixing of scalar field with curvature. The equations of motion for the field look quite unusual, namely

$$2\omega_{\rm BD}\tilde{\Box}\varphi = -\varphi\tilde{R} + (\tilde{\nabla}\varphi)^2\left(\frac{\omega_{\rm BD}}{\varphi} - \frac{\partial\omega_{\rm BD}}{\partial\varphi}\right) + 2M_{\rm Pl}^{-2}\varphi U', \tag{39}$$

and one can see that the field is sourced by the curvature. For convenience we can eliminate the Ricci scalar in favor of the trace of the energy–momentum tensor, which can be obtained by taking the trace of equation (38), resulting to

$$\tilde{\Box}\varphi = \frac{1}{3+2\omega_{\rm BD}}\left[M_{\rm Pl}^{-2}\tilde{T} - \frac{\mathrm{d}\omega_{\rm BD}}{\mathrm{d}\varphi}(\tilde{\nabla}\varphi)^2 + 2M_{\rm Pl}^{-2}(\varphi U' - 2U)\right]. \tag{40}$$

Since matter has no direct coupling with the scalar field in the Jordan frame, its energy–momentum tensor should be conserved. Indeed, using the evolution equation, it can be demonstrated that

$$\tilde{\nabla}_\mu\tilde{T}^{\mu\nu} = 0. \tag{41}$$

However, since the field gets entangled with curvature, its energy–momentum tensor is not conserved thereby the energy–momentum tensor of matter plus the energy–momentum tensor of the field is not a conserved quantity in the Jordan frame.

The equations of motion look quite complicated in the Jordan frame as the scalar degree of freedom is mixed with the curvature. It is therefore desirable to transform to a frame where the action (37) is diagonalized such that we have Einstein description along with a standard scalar field. Such a frame is known as Einstein frame. The transition to Einstein frame can be realized by the conformal transformation[172,173,197,198]

$$\tilde{g}_{\mu\nu} = A^2 g_{\mu\nu}, \tag{42}$$

where A is known as conformal factor and $g_{\mu\nu}$ is the Einstein metric. Conformal transformation scales the spacetime interval $\tilde{\mathrm{d}s}^2 = A^2 \mathrm{d}s^2$ and can be thought as local scale transformation. It is customary to use A^2 as we want to ensure that the pre-factor of $g_{\mu\nu}$ should be positive. Let us immediately note that

$$\tilde{g}^{\mu\nu} = A^{-2} g^{\mu\nu}; \quad \sqrt{-\tilde{g}} = A^4 \sqrt{-g}. \tag{43}$$

Since we want to find out the Ricci scalar in Einstein frame, we need to look for the transformation of Christoffel symbols, namely[198]

$$\tilde{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + (\Omega_\rho \delta^\mu_\nu + \Omega_\nu \delta^\mu_\rho - \Omega^\mu g_{\rho\nu}), \tag{44}$$

where $\Omega \equiv \ln A$, and we define $\Omega_\mu \equiv \partial_\mu \Omega$. Hence, we can now transform R to Einstein frame as

$$\tilde{R} = A^{-2}(R - 6\Box\Omega - 6 g^{\mu\nu} \Omega_\mu \Omega_\nu). \tag{45}$$

Note that the second term will not affect the equations of motion and can be dropped. We can then transform the action to Einstein frame

$$\begin{aligned} \mathcal{S} = & \int \sqrt{-g}\mathrm{d}^4x \left[\frac{M_{\mathrm{Pl}}^2}{2} R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right] \\ & + \int \sqrt{-g}\mathrm{d}^4x \mathcal{L}_m(\psi, A(\phi)^2(\phi) g_{\mu\nu}), \end{aligned} \tag{46}$$

provided that we make the following identifications:

$$\begin{gathered} \varphi = A^{-2}; \quad \left(\frac{\frac{\mathrm{d}A}{\mathrm{d}\phi}}{A} \right)^2 = \frac{1}{4\varphi^2} \left(\frac{\mathrm{d}\varphi}{\mathrm{d}\phi} \right)^2 = M_{\mathrm{Pl}}^{-2} \frac{1}{2(2\omega_{\mathrm{BD}} + 3)}; \\ V(\phi) = \frac{U(\varphi)}{M_{\mathrm{Pl}}^2 \varphi^2}, \end{gathered} \tag{47}$$

which define the field ϕ and its potential. Let us note that the action in Einstein frame (46) is diagonalized giving rise to standard Einstein equations plus a canonical field which is nonminimally coupled to matter. Hence all the complications of the Brans–Dicke Lagrangian are imbibed in the conformal coupling.

The evolution equations which follow from (46) are

$$G_{\mu\nu} = T_{\mu\nu} + T^{\phi}_{\mu\nu}, \tag{48}$$

$$\Box\phi = \frac{\alpha}{M_{\rm Pl}}T + \frac{{\rm d}V}{{\rm d}\phi} \Rightarrow V_{\rm eff} = V(\phi) + \frac{\alpha}{M_{\rm Pl}}\phi T; \quad \alpha \equiv M_{\rm Pl}\frac{{\rm d}\ln A(\phi)}{{\rm d}\phi}, \tag{49}$$

where $T \equiv T^{\mu}_{\mu} = 3p - \rho$ and α is the coupling that for simplicity might be considered to be a constant. For instance, $f(R)$ theories ($\omega_{\rm BD} = 0$) correspond to $\alpha = 1/\sqrt{6}$. Furthermore, note that from (48) we deduce that $T_{\mu\nu} + T^{\phi}_{\mu\nu}$ is conserved as expected. It is also important to mention that the effect of coupling in the effective potential becomes relevant in the nonrelativistic case as T vanishes in case of relativistic matter.

It is worth commenting on the relationship between Brans–Dicke parameter and the coupling constant α. Relations (47) and (49) tell us that $\alpha = 1/\sqrt{6}$ for $\omega_{\rm BD} = 0$ which corresponds to $f(R)$. In general, the coupling constant α is typically of the order of one, whereas local gravity constrains demand that $\omega_{\rm BD} \gg 6000$ thereby α is vanishingly small. In this case we are dealing with the trivial regime of scalar–tensor theories. It should be emphasized that if accelerated expansion takes place in this case, it is simply due to the flatness of the potential. In such cases one does not need the chameleon mechanism and the corresponding scalar theories are of little interest. We should also note that at the onset it follows from (37) that $G_{\rm eff} = A(\phi)G$. However, what one measures in Cavendish experiment is different and can be inferred, for instance, from weak field limit working in the Jordan frame, namely

$$G_{\rm eff} = GA(\phi)(1 + 2\alpha^2). \tag{50}$$

It is illuminating to quote the relationship in the Einstein frame as

$$G_{\rm eff} = G(1 + 2\alpha^2), \tag{51}$$

where the second term in the expression within the parenthesis is due to the exchange of the scalar degree of freedom which is obviously absent in the case of minimal coupling. Modification of gravity *à la* scalar-tensor theory (under consideration) is reduced to spin-2 object along with a scalar degree of freedom which couples to matter with strength of the order of the gravitational coupling. The latter would be in sharp contradiction with the solar physics where Einstein theory has phenomenal accuracy. Thus, the scalar degree of freedom needs to be suppressed or be screened out locally.

As we mentioned above, since the field does not directly couple to matter in Jordan frame, the energy–momentum tensor in this frame is conserved, namely

$$\tilde{\nabla}^{\mu}\tilde{T}_{\mu\nu} = 0. \tag{52}$$

In order to check the conservation of the matter energy–momentum tensor in the Einstein frame, we mention that it transforms while passing from Jordan to Einstein frame as

$$\tilde{T}_{\mu\nu} = -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta\mathcal{S}_m}{\delta\tilde{g}^{\mu\nu}} = A^{-2}(\phi)T_{\mu\nu}. \tag{53}$$

Acting with the $\tilde{\nabla}$ operator on Eq. (53) we obtain

$$\tilde{\nabla}^\mu T_{\mu\nu} - 2\frac{\alpha}{M_{\rm Pl}}A^{-2}T_{\mu\nu}\partial^\mu\phi = 0, \tag{54}$$

where

$$\tilde{\nabla}^\mu T_{\mu\nu} = A^{-2}g^{\mu\rho}(\partial_\rho T_{\mu\nu} - \tilde{\Gamma}^\sigma_{\mu\rho}T_{\sigma\nu} - \tilde{\Gamma}^\sigma_{\nu\rho}T_{\mu\sigma}). \tag{55}$$

Using the expression (44) for the Christofell symbols in Jordan frame and Eq. (54), we finally arrive at

$$\nabla^\mu T_{\mu\nu} = \frac{\alpha}{M_{\rm Pl}}T\partial_\nu\phi. \tag{56}$$

Thus, from the conservation of the total energy–momentum tensor $T_{\mu\nu} + T^\phi_{\mu\nu}$, we deduce the conservation for the scalar field, namely

$$\nabla^\mu T^\phi_{\mu\nu} = -\frac{\alpha}{M_{\rm Pl}}T\partial_\nu\phi. \tag{57}$$

One of the most important implications of conformal transformations is related to the transformation of particle masses frame, namely the particle masses become field dependent in the Einstein frame. Indeed,

$$\tilde{T}^{\mu\nu} = \int\frac{\tilde{m}}{\sqrt{-\tilde{g}}}\frac{{\rm d}z^\mu}{{\rm d}\tilde{s}}\frac{{\rm d}z^\nu}{{\rm d}\tilde{s}}\delta(z - x(s)){\rm d}\tilde{s} = A^{-6}\int\frac{A\tilde{m}}{\sqrt{-g}}\frac{{\rm d}z^\mu}{{\rm d}s}\frac{{\rm d}z^\nu}{{\rm d}s}\delta(z - x(s)), \tag{58}$$

which using the transformation of energy–momentum tensor allows us to identify the particle mass in the Einstein frame as

$$m = A(\phi)\tilde{m}. \tag{59}$$

Let us take the example of FRW cosmology and check for the conformal equivalence[194]

$${\rm d}s^2 = a^2(\tau)[{\rm d}\tau^2 - ({\rm d}x^2 + {\rm d}y^2 + {\rm d}z^2)]; \quad {\rm d}t = a(t){\rm d}\tau. \tag{60}$$

Thus, the FRW metric is conformally mapped to Minkowski metric through

$$g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu} \equiv \varphi\tilde{g}_{\mu\nu}. \tag{61}$$

The Einstein–Hilbert action along with matter part transforms to

$$\mathcal{S} = -\frac{3}{4}M_{\rm Pl}^2\int\frac{(\tilde{\nabla}\varphi)^2}{\varphi}{\rm d}^4x + \mathcal{S}_{\rm m}(\varphi\tilde{g}_{\mu\nu}, \psi), \tag{62}$$

which is the action of a free scalar field plus a matter part in Minkowski spacetime. In this case, the equation of motion for the field is[194]

$$\frac{3}{4}\frac{\varphi'^2}{\varphi} = M_{\rm Pl}^{-2}\tilde{\rho}, \tag{63}$$

where from now on a prime denotes derivative with respect to the conformal time τ.

Second, since φ explicitly enters in the matter action, its energy–momentum tensor is not conserved, namely

$$\tilde{\nabla}^{\mu}\tilde{T}_{\mu\nu} = \frac{\partial_{\nu}\varphi}{2\varphi}\tilde{T}, \tag{64}$$

which leads to the following equation[194]:

$$\tilde{\rho}' = \frac{\varphi'}{2\varphi}(\tilde{\rho} - 3\tilde{p}). \tag{65}$$

We emphasize that in Minkowski spacetime we are left with an evolving field φ which is coupled to matter, plus the particle masses also evolve with the evolution of the field. The latter contains the total information of FRW dynamics. Indeed, one can readily verify that Eqs. (63) and (65) are equivalent to the equations of standard cosmology. Noting that $H(t) = \varphi'/2\varphi^{3/2}$, we find that

$$H^2 = \frac{\rho}{3M_{\rm Pl}^2}; \quad \dot{\rho} + 3H(\rho + p) = 0 \tag{66}$$

in the Einstein frame, where we changed from conformal to cosmic time and used the transformation law $\tilde{T}_{\mu\nu} = \varphi T_{\mu\nu}$. Second, one might wonder what could lead to redshift in the flat spacetime, which is static and the field has no coupling to radiation. In fact, the evolution of masses mimics the redshift effect in Minkowski spacetime. Let us consider the frequency radiated during an atomic transition in a distant galaxy at time t:

$$\nu(t) = \frac{1}{2}\tilde{m}^2\alpha_{\rm F}^2\left(\frac{1}{n'^2} - \frac{1}{n^2}\right), \tag{67}$$

where $\tilde{m}$ is the electron mass and $\alpha_{\rm F}$ is the hyperfine structure constant. Its ratio with the frequency observed by an observer on earth at the present epoch is given by

$$\frac{\nu_0}{\nu(t)} = \frac{\tilde{m}_0}{\tilde{m}(t)} = \frac{a_0}{a} = (1 + z). \tag{68}$$

We should note that ν_0 is the frequency emitted today whereas $\nu(t)$ is its counterpart emitted earlier at cosmic time t when mass of electron was $m(t) < m_0$. Since we are in Minkowsky space time, $\nu(t)$ is observed today with the numerical value it was emitted at time t. Hence, (68) mimics the redshift effect correctly. One can further try to understand the thermal history in Minkowski spacetime which is filled with microwave background radiation with temperature equal to 2.7 K. In particular, to understand in this frame the radiation–matter decoupling, the synthesis of light elements, and the big bang itself. Similarly to the previous discussion, the key feature here is attributed to the evolution of masses of elementary particles. The radiation matter equilibrium corresponds to the epoch when,

$$|E_{\rm BE}(t)| = \frac{1}{2}\frac{\tilde{m}(t)}{\tilde{m}_0}\tilde{m}_0\alpha_F^2 = \frac{\tilde{m}(t)}{\tilde{m}_0}13.6\,{\rm eV} \lesssim 10^{-4}\,{\rm eV} \to \tilde{m}(t) \lesssim 10^{-5}\tilde{m}_0, \tag{69}$$

where $E_{\rm BE}(t)$ is the binding energy of hydrogen atom at cosmic time t. Here we used the condition $E_{\rm BE}(t_0) < 2.7\,{\rm K} \simeq 10^{-4}\,{\rm eV}$ for equilibrium, while Big Bang obviously corresponds to the epoch when $\tilde{m}(t) = 0$.

It is also possible to reproduce the local physics in flat spacetime.[194] In fact, one can go ahead and verify the same at the level of perturbations in the case of FRW spacetime.[195] To sum up, in the above discussion we have tried to convince the reader that the relation between physical observables is the same in all frames connected to each other by a conformal transformation.

Being convinced by the conformal equivalence, let us consider the case of coupling to standard matter (cold dark matter+baryonic matter). As the universe enters the matter-dominated era, the nonminimal coupling builds up:

$$V_{\rm eff} = V(\phi) + \frac{\alpha}{M_{\rm Pl}}\rho_{\rm m}\phi, \tag{70}$$

giving rise to minimum of the effective potential such that the minimum itself evolves with $\rho_{\rm m}$. As mentioned in the Introduction, we are interested in the scaling behavior in the post inflationary era, which implies that the potential should mimic the steep exponential potential $V(\phi) = V_0 e^{-\lambda\phi/M_{\rm Pl}}$. The dynamical investigation in this case shows that we have scaling solution, which is accelerated,[178] and we obtain an equation of state of the form

$$w_\phi = -\frac{\alpha}{\alpha + \lambda}, \tag{71}$$

which implies that there is a de Sitter attractor for $\alpha \gg \lambda$. Let us note that matter now does not evolve with $w_{\rm m} = 0$ but rather with (71). This is a scaling solution which is accelerating for large value of coupling ($\alpha > \lambda/2$). In case of minimally coupled scalar field with $\alpha = 0$, it obviously reduces to standard scaling solution (see Sec. 3.3 for details).

At the onset it might sound a required arrangement but there is a serious drawback. Soon after the universe enters the matter-dominated era, the attractor is reached destroying the matter era. It is more than desirable that the matter era be left intact.

There is still a way out, namely to construct a scenario in which the standard matter does not couple to the field but massive neutrino matter does. Neutrinos with masses around 1 eV turn nonrelativistic around the present epoch giving rise to non-zero T, thus inducing a minimum in the effective potential. This arrangement leaves the matter era unchanged. The required Einstein action has the following form

$$\begin{aligned}\mathcal{S} = &\int \sqrt{-g}\,{\rm d}^4x \left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right] \\ &+ \int \sqrt{-g}\,{\rm d}^4x[\mathcal{L}_{\rm m}(\psi, A^2(\phi)g_{\mu\nu}) + \mathcal{L}^\nu{}_{\rm m}(\psi, A^2(\phi)g_{\mu\nu})],\end{aligned} \tag{72}$$

where $\mathcal{L}^\nu_{\rm m}$ is the action of neutrino matter. We mention that the arrangement in action(72) implies nonminimal coupling of standard matter in the Jordan frame, namely $\mathcal{L}_{\mathrm{m}}(A^{-2}(\phi)g_{\mu\nu})$, such that the conformal transformation to Einstein frame leaves standard matter minimally coupled in the Einstein frame. In this case, the effective potential becomes

$$V_{\mathrm{eff}} = V(\phi) + \frac{\alpha}{M_{\mathrm{Pl}}}\rho_{\mathrm{m}}^{\nu}\phi, \tag{73}$$

which in case of Type II models can trigger the exit from scaling regime to late-time acceleration. We shall make use of this mechanism in Sec. 3.2.

2.4. *Instant preheating*

As mentioned in the Introduction, the models of quintessential infation belong to the category of *nonoscillatory* models, and thus the conventional reheating mechanism is not applicable to them. One natural and universal mechanism of reheating is provided by gravitational particle production. After inflation, the geometry of spacetime undergoes a nonadiabatic change giving rise to particle production, which could reheat the universe. Unfortunately, this process is extremely inefficient. The way out is provided by an alternative mechanism dubbed instant preheating studied in Refs. 49, 50, 169–171. The method relies on the assumption that the inflaton ϕ interacts with another scalar field χ, which is coupled to the fermionic field via Yukawa-type of interaction. Supposing that inflation ends when $\phi = \phi_{\mathrm{end}}$, we can shift the field $\phi \to \phi' = \phi - \phi_{\mathrm{end}}$ such that inflation ends at the origin, and call the new field ϕ. The Lagrangian is written as

$$\mathcal{L}_{\mathrm{int}} = -\frac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\chi, \tag{74}$$

where the couplings are supposed to be positive with $g, h < 1$ in order for the perturbation treatment to be valid. The χ field does not possess a bare mass, while its effective mass depends upon ϕ as

$$m_\chi = g\phi. \tag{75}$$

In the models under consideration, as discussed in the Introduction, inflation ends in the regime where the field potential is represented by a steep exponential function, so that the field ϕ soon enters the kinetic regime after the end of inflation. In this case, the field would enter into fast-roll running away from the origin. Hence, production of χ particle after inflation can take place if m_χ changes nonadiabatically as

$$\dot{m}_\chi \gtrsim m_\chi^2 \to \dot{\phi} \gtrsim g\phi^2. \tag{76}$$

Condition (76) implies that

$$|\phi| \lesssim |\phi_p| = \left(\frac{\dot{\phi}_{\mathrm{end}}}{g}\right)^{1/2}. \tag{77}$$

In order to estimate $\dot{\phi}$, we assume slow roll to hold till the end of inflation. In case of single-field inflation taking place in four-dimensional spacetime and braneworld cosmology, respectively we have

$$H^2 \simeq \frac{V}{3M_{\rm Pl}^2}; \quad H^2 \simeq \frac{1}{6M_{\rm Pl}^2}\frac{V^2}{\lambda_{\rm b}}, \tag{78}$$

where $\lambda_{\rm b}$ is the brane tension. Using then the slow-roll equation for the field $-3H\dot{\phi} \simeq V'$, in both cases we find that

$$|\dot{\phi}_{\rm end}| \simeq V_{\rm end}^{1/2}\epsilon_{\rm end}^{1/2} = V_{\rm end}^{1/2}, \tag{79}$$

where $\epsilon = \epsilon_0 \frac{4\lambda_{\rm b}}{V}$ in braneworlds with $\epsilon_0 \simeq (M_{\rm Pl}^2/2)(V'/V)^2$ is the standard slow-roll parameter. We can now estimate the field where production of χ particles takes place

$$\phi \lesssim \phi_{\rm pd} \simeq \left(\frac{V_{\rm end}^{1/2}}{g}\right)^{1/2} \to g^2 \gtrsim M_{\rm Pl}^{-4} V_{\rm end} \quad (\phi_{\rm pd} \lesssim M_{\rm Pl}). \tag{80}$$

Let us make a very crude estimate. Assuming that BICEP2 data are correct, that is ignoring the dust discussion that is currently taking place in the literature,[196] it is implied that the scale of inflation is $H_{\rm in} \sim 10^{-2}M_{\rm Pl}$. As for $H_{\rm end}$, it differs from $H_{\rm in}$ and it may be less by two orders of magnitudes depending upon the model. Anyway, assuming $H_{\rm end} \sim 10^{-2}M_{\rm Pl}$, we find that $g \lesssim 0.1$, though this range is narrower in practice thereby production takes place in a small neighborhood around $\phi = 0$. We can also estimate the production time as

$$t_{\rm pd} \simeq \frac{\phi}{|\dot{\phi}|} \simeq g^{-1/2}|\dot{\phi}_{\rm end}|^{-1/2} \to t_{\rm pd} \simeq H_{\rm end}^{-1}, \tag{81}$$

which is very small implying that particle production commences soon after inflation ends.

As a next step, we will estimate the χ-particle occupation number. To this effect, we use the uncertainty relation to obtain the estimation for the wave number, which allows us to extract the occupation number for χ particles[154,170] as

$$k_{\rm pd} \simeq t_{\rm pd}^{-1} \simeq \sqrt{g|\dot{\phi}_{\rm end}|} \to n_k \sim {\rm e}^{-\pi k^2/k_p^2}. \tag{82}$$

Thus, the number density of χ-particles is

$$N_\chi = \frac{1}{(2\pi)^3}\int_0^\infty n_k {\rm d}^3\mathbf{k} = \frac{(g|\dot{\phi}_{\rm end}|)^{3/2}}{(2\pi)^3}, \tag{83}$$

while the energy density of the created particles reads as

$$\rho_\chi = N_\chi m_\chi = \frac{(g|\dot{\phi}_{\rm end}|)^{3/2}}{(2\pi)^3} g|\phi_p| = \frac{g^2 V_{\rm end}}{(2\pi)^3}. \tag{84}$$

If the particle energy produced at the end of inflation is supposed to be thermalized, using Eqs. (78) and (84) we find that

$$\left(\frac{\rho_\phi}{\rho_{\rm r}}\right)_{\rm end} \simeq \frac{(2\pi)^3}{g^2}. \tag{85}$$

This is an important formula which can be used to set a limit on the temperature of radiation, and therefore to control the duration of the kinetic regime. Then, using (85) would give the lower limit on the coupling g.

At this point let us mention that the energy of χ particles redshifts as a^{-3}, and can backreact on the evolution. In order to avoid this problem we should enforce these particles to decay very fast after their creation. Since ϕ runs fast after inflation has ended, the mass of χ grows larger making it to decay into $\bar{\psi}\psi$, with the corresponding decay width given by

$$\Gamma_{\bar{\psi}\psi} = \frac{h^2 m_\chi}{8\pi} = \frac{h^2}{8\pi} g|\phi|. \tag{86}$$

Indeed, the decay rate is larger for larger values of m_χ. Hence, the requirement that the decay of χ into fermions is completed before their backreaction on the post inflationary dynamics becomes important, imposes a bound on the numerical value of the coupling h. In particular

$$\Gamma_{\bar{\psi}\psi} \gg H_{\mathrm{end}} \Rightarrow h^2 \gtrsim 8\pi \frac{H_{\mathrm{end}}}{g|\phi|} = \frac{8\pi}{\sqrt{3}} \frac{V_{\mathrm{end}}}{g|\phi| M_{\mathrm{Pl}}}, \tag{87}$$

which provides the lower bound on the numerical values of h. In realistic models of quintessential inflation we find a wide parameter range (g, h) that can give rise to the required preheating. This mechanism is quite efficient and can easily circumvent the aforementioned problem related to excessive production of gravity waves. Eq. (85) is the main result of this subsection, which shall be used to fix the radiation temperature at the end of inflation in accordance with the nucleosynthesis constraint.

2.5. *Relic gravitational waves*

One of the important predictions of the inflationary paradigm is the production of gravitational waves that are generated quantum-mechanically during inflation. These gravitational waves induce polarization of the microwave background radiation, such that the size of the effect depends upon their amplitude. The confirmation of recent B mode polarization measurements could emerge as a strong direct observational support of inflation.

Gravitation waves in a spatially homogeneous and isotropic spacetime are small tensor perturbations around the background[34,35,37,38,45,46,48,174]

$$ds^2 = a^2(\tau)(d\tau^2 - a^2(\delta_{ij} + h_{ij}) dx^i dx^j), \tag{88}$$

which are transverse and traceless, namely $\partial_i h^{ij} = 0; h^i_i = 0$, leaving behind two degrees of freedom. The Einstein equations then imply the Klein–Gordon equation for tensor perturbations:

$$\Box h_{ij} = 0 \rightarrow \phi'_k + 2\frac{a'}{a} + k^2 \phi_k; \quad (h_{ij} \sim \phi_k(\tau) \mathrm{e}^{-i\mathbf{kx}} e_{ij}), \tag{89}$$

where e_{ij} is the polarization tensor and $k = 2\pi a/\lambda$ is the comoving wave number, while "′" denotes the derivative with respect to conformal time $a(\tau)d\tau = dt$.

Equation (89) can be transformed to a convenient form in terms of a new function, $\mu_k(\tau) \equiv a(\tau)\phi_k(\tau)$, as

$$\mu_k''(\tau) + \left[k^2 - \frac{a''(\tau)}{a(\tau)}\right]\mu_k(\tau) = 0, \tag{90}$$

which resembles Schrodinger equation with time-dependent potential $U = a''/a$. In the following we assume inflation to be de Sitter, in which case $\tau = -[H_{\rm dS}a(\tau)]^{-1}$. It is important to distinguish two regimes, namely

$$k\tau \ll 1 \Rightarrow \frac{k}{aH} \gg 1 \quad \text{modes outside the Hubble radius or Horizon}, \tag{91}$$

$$k\tau \gg 1 \Rightarrow \frac{k}{aH} \ll 1 \quad \text{modes inside the Hubble radius or Horizon}. \tag{92}$$

Let us first illustrate the underlying idea using a heuristic argument. The case $k^2 \ll U$ implies that modes are outside the horizon. As $U \sim 1/\tau^2$ the equation of motion (89) has a simple solution in this regime

$$\phi_k \simeq C_1 + C_2 \int \frac{\mathrm{d}\tau}{a^2(\tau)}, \tag{93}$$

where the second term becomes smaller and smaller as the universe expands nearly exponentially during inflation, and therefore the perturbations freeze to a constant value outside the horizon or on super-horizon scales. On the other hand, deep inside the horizon or in the sub-Hubble limit, Eq. (89) reduces to the equation of simple harmonic motion giving rise to oscillating solution $\mu_k(\tau) \sim \mathrm{e}^{\pm ik\tau}$. In this limit curvature effects are negligible. The choice of positive frequency modes in this limit defines the vacuum state named Bunch–Device vacuum.

In the standard scenario inflation is followed by radiative regime, whereas in models of quintessential inflation it is the kinetic regime that commences after inflation (followed by the radiation era). Let us suppose that the transition occurs at $\tau = \tau_*$. Thus, the solution whose asymptotes we have just described is valid in the regime $-\infty < \tau < \tau_*$. Indeed, the exact solution of (90) in this case, corresponding to "in" state, is given by

$$\phi_{k(\rm in)} = \frac{1}{a(\tau)}\left(1 - \frac{i}{k\tau}\right)\mathrm{e}^{-ik\tau}, \quad -\infty < \tau < \tau_*, \tag{94}$$

where we have chosen the positive frequency solution in the sub-Hubble limit. At the transition point, which happens almost instantaneously at $\tau = \tau_*$, the space-time curvature changes abruptly giving rise to particle production. The solution of Eq. (90) in the new phase, corresponding to "out" state, also acquires the negative frequency component

$$\phi_{k(\rm out)} = \frac{1}{a(\tau)}(\alpha_k e^{-ik\tau} + \beta_k \mathrm{e}^{ik\tau}), \quad \tau > \tau_*, \tag{95}$$

where α_k and β_k are the Bogoliubov coefficients. Solution (95) is valid till the new cosmic phase transition takes place. The occupation number of particles produced in this process is given by

$$n_k = |\beta_k|^2. \tag{96}$$

Let us note that the nonadiabatic character of the process is essential for particle production. Within a given phase, spacetime curvature is felt less significantly when changes are adiabatic, such that $\beta_k \sim 0$, which is similar to Minkowski spacetime where vacuum is invariant under Poincare transformations. In the case of phase transitions, as the vacuum state evolves across the transition point, it no longer remains empty. We mention that in curved spacetime the vacuum state in general does not remain empty at later times. However, the occupation number of particles created within a given phase is negligible.

The Bogoliubov coefficients can be determined by demanding the continuity of ϕ_k and ϕ_k' at the transition point. Following this procedure, one can find out β_k corresponding to transitions: kinetic to radiation, radiation to matter and matter to dark energy. However, the transition from inflation to post inflationary phase is the most prominent. In scenarios of quintessential inflation, the kinetic regime essentially follows inflation, and we shall be interested in computing the energy density of gravitational waves generated across this transition, ignoring the contributions from other transitions.

In the preceding discussion we have indicated the solution of Eq. (90) for de Sitter phase. In fact, the exact solution of (90) for a power-law-type post-transition expansion can be expressed through Hankel function. In general

$$a = \left(\frac{t}{t_0}\right)^n = \left(\frac{\tau}{\tau_0}\right)^{\frac{1}{2}-c}, \quad n = \frac{2}{3(1+w)}, \quad c = \frac{3}{2}\left(\frac{w-1}{3w+1}\right). \tag{97}$$

For de Sitter and kinetic regimes, in which we are interested, $c = 3/2, a(\tau) = \tau/\tau_0$ & $c = 0, a(\tau) = (\tau/\tau_0)^{1/2}$ respectively. Before the transition to kinetic regime, the system is in the adiabatic vacuum or "in" state given by (94). The "out" state contains both positive and negative frequency modes,

$$\phi_{k(\mathrm{out})} = \alpha_k \phi^+_{\mathrm{out}}(k\tau) + \beta_k \phi^-_{\mathrm{out}}(k\tau), \tag{98}$$

where positive (negative) frequency modes are given by

$$\phi^{+,-}_{\mathrm{out}} = \left(\frac{\pi\tau_0}{4}\right)^{1/2} H^{(2,1)}_{(0)}(k\tau), \tag{99}$$

with $H^{(2)}_{(0)} = (H^{(1)}_{(0)})^*$. In order to find β_k we need to incorporate the matching of the solution across the transition. On super-horizon scales $\phi_{k(\mathrm{in})}$ freezes to a constant value, whereas on the other side (inside the horizon, in the kinetic regime) we can take the small k limit of the Hankel function and then match the "in" and "out" solutions. The matching gives[48]

$$\beta_k \simeq \frac{1}{2\pi}\left(k\tau_{\mathrm{kin}}\right)^{-3}. \tag{100}$$

Following Refs. 33, 35 and 48 we can then compute the energy density of gravitational waves produced during the transition from inflation to kinetic regime as

$$\rho_{\rm g} = \frac{1}{\pi^2 a^4}\int k^3|\beta_k|^2 dk. \tag{101}$$

Additionally, the spectral energy density is defined as

$$\tilde{\rho}_{\rm g} \equiv \frac{\rm d}{{\rm d}\log k}\rho_{\rm g} = \frac{1}{\pi^2 a^4}(k^4|\beta_k|^2). \tag{102}$$

Hence, using Eqs. (100) and (101), we find that the energy density of gravitational waves produced during the transition under consideration is given by

$$\rho_{\rm g} = \frac{4}{3\pi^2}H_{\rm in}^2\rho_{\rm b}\left(\frac{\tau}{\tau_{\rm kin}}\right) = \frac{32}{3\pi}h_{\rm GW}^2\rho_{\rm b}\left(\frac{\tau}{\tau_{\rm kin}}\right), \tag{103}$$

where $h_{\rm GW}$ is the dimensionless amplitude of gravitational waves and $H_{\rm in}$ is the Hubble parameter at the commencement of inflation. $\rho_{\rm b}$ stands for the background energy density, which consists of the inflaton energy density in the kinetic regime plus the energy density of radiation created by an alternative mechanism. The radiation energy density is negligible as compared to that of the inflaton in the beginning, but eventually it dominates since it redshifts slower (a^{-4}) than the inflaton energy density (which redshifts as a^{-6} in the kinetic regime). The duration of kinetic regime depends upon the temperature of radiation created after inflation. At the commencement of radiative regime we have

$$\rho_{\rm g}(\tau = \tau_{\rm eq}) = \frac{64}{3\pi}h_{\rm GW}^2\rho_{\rm r}\left(\frac{T_{\rm kin}}{T_{\rm eq}}\right)^2. \tag{104}$$

We remind that in the scenarios of quintessential inflation the post inflationary dynamics is governed by steep potential, such that the kinetic regime is fast reached after inflation has ended. In order to estimate the order of magnitude, it is a good approximation to assume that $T_{\rm kin} \simeq T_{\rm end}$, and thus

$$1 = \left(\frac{\rho_\phi}{\rho_{\rm r}}\right)_{\rm eq} = \left(\frac{\rho_\phi}{\rho_{\rm r}}\right)_{\rm kin}\left(\frac{a_{\rm kin}}{a_{\rm eq}}\right)^2 \Rightarrow \left(\frac{T_{\rm kin}}{T_{\rm eq}}\right) \simeq \left(\frac{\rho_\phi}{\rho_{\rm r}}\right)_{\rm end}^{1/2}. \tag{105}$$

Substituting this expression into Eq. (104) we obtain

$$\left(\frac{\rho_{\rm g}}{\rho_{\rm r}}\right)_{\rm eq} = \frac{64}{3\pi}h_{\rm GW}^2\left(\frac{\rho_\phi}{\rho_{\rm r}}\right)_{\rm end}. \tag{106}$$

This is an important result which allows us to impose observational constraint, namely the nucleosynthesis constraint ($(\rho_{\rm g}/\rho_{\rm r})_{\rm eq} \lesssim 0.01$), on the radiation energy density produced at the end of inflation:

$$\left(\frac{\rho_{\rm r}}{\rho_\phi}\right)_{\rm end} \gtrsim 10^2 \times \frac{64}{3\pi}h_{\rm GW}^2. \tag{107}$$

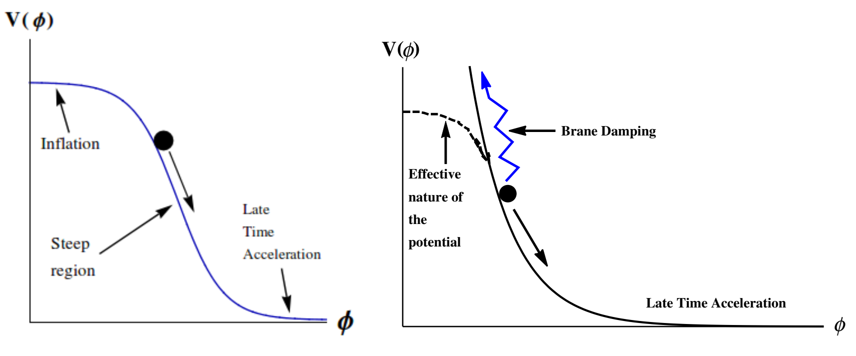
Equation (107) provides the lower bound on the radiation temperature at the end of inflation, thereby it restricts the duration of the kinetic regime. Smaller temperature values would result in longer kinetic regime and enhancement of energy in the gravitational waves, that would conflict with the nucleosynthesis constraint. Let us note that the energy density produced in the process of gravitational particle production ($\rho_{\rm r} = 0.01 \times g_p H_{\rm end}^4$, g_p being the number of degrees of freedom produced that typically varies between 10 and 100) falls short to meet the above requirement. If we invoke the crude estimate $h_{\rm GW}^2 \simeq H_{\rm end}^2/8\pi$, we deduce that we miss the nucleosynthesis constraint by several orders of magnitude. In case of instant preheating, Eq. (107) would place a bound on the numerical value of the coupling g ($g \gtrsim H_{\rm end}/M_{\rm Pl}$), though the actual numbers are model dependent.

3. Quintessential Inflation at Last

In the preceding section we briefly described the concepts needed to unify early and late time phases of cosmic acceleration using a single scalar field. Although it is usually easy to integrate late time acceleration and thermal history, the problem often arises while reconciling the inflationary description with observational constraints. As mentioned in the Introduction, unification of quintessence and dark energy in general requires a scalar field with potential which is shallow at early times, followed by steep exponential-like behavior till late times, where it again turns shallow, as shown in Fig. 2(a). These generic potentials come into two classes: Type I includes subclasses of potentials which are steep for most of the universe history and shallow at late times. Type II subclass can facilitate slow roll at early times, followed by steep behavior thereafter. We shall first describe quintessential inflation in models of Type I.

3.1. *Quintessential inflation on the brane*

In Randall–Sundrum braneworld scenario, our four-dimensional spacetime (brane) is assumed to be embedded into a five-dimensional AdS bulk, with matter living on the brane. The effective Einstein equations on the brane, obtained by projecting the



(a) (b)

Fig. 2. Schematic diagrams of desired potentials in models of quintessential inflation. (a) A typical potential of subclass Type II. (b) A typical potential of subclass Type I.

bulk dynamics on the brane, contain high energy corrections, quadratic in energy–momentum tensor. As a result, the Friedmann equation on the brane acquires quadratic dependence in matter density:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\rho\left(1+\frac{\rho}{2\lambda_{\rm b}}\right), \tag{108}$$

where ρ is the total matter energy density on the brane, which reduces to the field energy density in case of inflation. Equation (108) implies that the Hubble damping in the field equation on brane, namely in

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \tag{109}$$

becomes large in the high energy limit $\rho_\phi \gg \lambda_{\rm b}$. This feature might facilitate slow roll of the field along the steep potential on the brane (see Fig. 3 for the effective nature of the potential during inflation). Indeed, the slow roll parameters in this case modify to

$$\epsilon = \epsilon_0 \frac{1+\dfrac{V}{\lambda_{\rm b}}}{\left(1+\dfrac{{\rm s}V}{2\lambda_{\rm b}}\right)^2}; \quad \eta = \frac{\eta_0}{1+\dfrac{V}{2\lambda_{\rm b}}}, \tag{110}$$

where ϵ_0 and η_0 are the standard slow roll parameters. In the high energy limit $V \gg \lambda_{\rm b}$, where brane corrections are important, the slow roll parameters reduce to

$$\epsilon \simeq 4\epsilon_0 \frac{\lambda_{\rm b}}{V}; \quad \eta \simeq 2\eta_0, \frac{\lambda_{\rm b}}{V}, \tag{111}$$

which imply that $\epsilon, \eta \ll 1$ in the high energy limit even if ϵ_0, η_0 are not small, i.e. even if the potential is steep. Hence, high energy brane corrections can indeed give rise to slow roll along a steep exponential potential of the form

$$V(\phi) = V_0 {\rm e}^{-\lambda\phi/M_{\rm Pl}}. \tag{112}$$

In this case, $\lambda_{\rm b}$, $V_{\rm end}$, and the potential value at the commencement of inflation $V_{\rm in}$, are related as

$$V_{\rm end} = 2\lambda^2\lambda_{\rm b}; \quad \frac{V_{\rm end}}{V_{\rm in}} = \mathcal{N}+1. \tag{113}$$

The COBE normalization then allows to determine $\lambda_{\rm b}$ and $V_{\rm end}$ in terms of the number of e-foldings as

$$\lambda_{\rm b} \simeq \frac{(8\pi)^4}{\lambda^4} \times 10^{-10}\left(\frac{M_{\rm Pl}}{\mathcal{N}}\right)^4 \tag{114}$$

$$V_{\rm end} \simeq 5\frac{(8\pi)^4}{\lambda^4} \times 10^{-10}\left(\frac{M_{\rm Pl}}{\mathcal{N}}\right)^4. \tag{115}$$

In this case, the spectral index n_S and tensor-to-scalar ratio r are given by

$$n_{\rm s} - 1 = -\frac{4}{\mathcal{N}}; \quad r = \frac{24}{\mathcal{N}}, \tag{116}$$

which gives $r = 0.4$ for $\mathcal{N} = 60$. In this model, inflation gracefully ends as field rolls down its potential and high energy corrections disappear. After inflation, we have steep exponential potential for which scaling solution is an attractor. Furthermore, the slope of the potential is determined by nucleosynthesis constraints,[107] namely

$$\Omega_\phi = 3\frac{(1+w_r)}{\lambda^2} \lesssim 0.01 \to \lambda \gtrsim 20, \tag{117}$$

which ensures that the scalar degree of freedom is adequately suppressed.

3.2. *Late time evolution*

There are several ways to obtain tracking behavior in models under consideration. For instance, a double exponential potential under specific conditions gives rise to the desired behavior. A cosh potential of the form $V = V_0(\cosh \tilde{\lambda}\phi/M_{\mathrm{Pl}} - 1)^p$ ($\lambda = p\tilde{\lambda}$), acquires exponential form for large value of its argument and reduces to power-law form, $V \sim (\phi/M_{\mathrm{Pl}})^{2p}$, around the origin. Consequently, the average equation of state parameter, $\langle w_\phi \rangle = (p-1)/(p+1)$, can take a desired value for a given numerical value of p. Another example is provided by the already mentioned inverse power-law potentials. In Fig. 3, we depict the evolution from the end of inflation to late-time cosmic acceleration. Although post-inflationary dynamics is satisfactory in models of Type I, unfortunately the description of inflationary phase itself is ruled out by the tensor-to-scalar ratio observations, since it proves to be too large. An attempt to lower the value of r was made by invoking a Gauss–Bonnet term in the bulk, however the modified equations of motion lead to moderate improvement and fails to meet the requirement even of BICEP2. No other way of resolution of this problem is known at present.

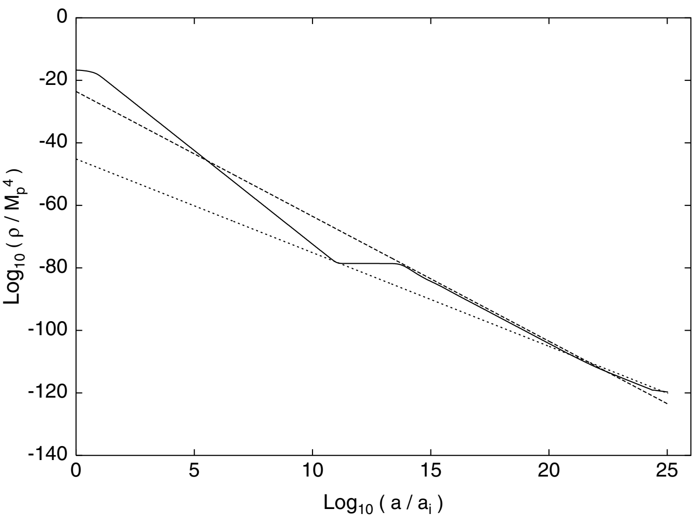


Fig. 3. Evolution of energy densities, in braneworld cosmology, from the end of inflation to the present epoch (see Ref. 48).

3.3. *Quintessential inflation in four-dimensional space time*

As mentioned above, Type II models can be used to implement the idea of unification in the standard FRW background (see Fig. 4(a) for a typical potential of this subclass). In this case we need to exit from scaling regime to late-time acceleration, which can be accomplished by invoking a nonminimal coupling of the field to massive neutrino matter[202,203] (see also[146,168,199–201,204–214]). When massive neutrinos turn nonrelativistic around the present epoch, their energy density gets directly coupled to the field, which triggers a minimum in the potential, where the field can settle giving rise to late-time acceleration (see Fig. 3). This can be realized in the variable gravity framework.[168,215–219]

In this scenario all elementary particles are directly coupled to a noncanonical scalar field in the Jordan frame (see Appendix A), while in the Einstein frame only massive neutrino matter has a direct coupling to the scalar field, whereas standard matter does not "see" it. The desired action in Einstein frame has the following form[146,168]:

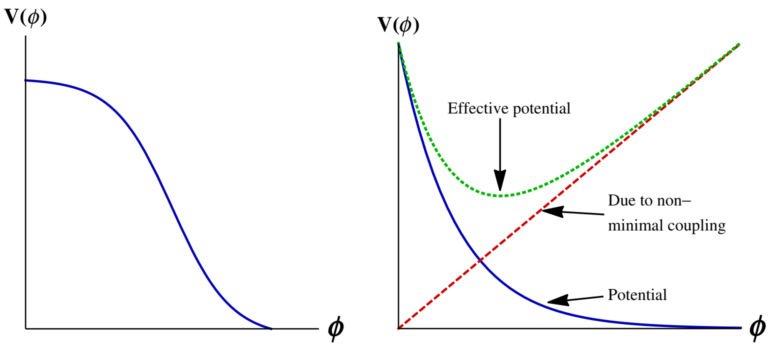
$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{k^2(\phi)}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right] + \mathcal{S}_m + \mathcal{S}_r + \mathcal{S}_\nu(\mathcal{C}^2(\phi)g_{\alpha\beta};\Psi_\nu), \tag{118}$$

with

$$k^2(\phi) = \left(\frac{\alpha^2-\tilde{\alpha}^2}{\tilde{\alpha}^2}\right)\frac{1}{1+\beta^2\mathrm{e}^{\alpha\phi/M_{\rm Pl}}} + 1, \tag{119}$$

$$V(\phi) = M_{\rm Pl}^4\mathrm{e}^{-\alpha\phi/M_{\rm Pl}}, \tag{120}$$

$$\mathcal{C}(\phi)^2 = \zeta\mathrm{e}^{2\tilde{\gamma}\alpha\phi/M_{\rm Pl}}. \tag{121}$$



(a) (b)

Fig. 4. (a) Schematic representation of a typical Type II potential, shallow at early times and steep thereafter. (b) The nonminimal coupling to neutrino matter induces a minimum in the (post inflationary) effective run-away potential of the scalar field.

In these expressions $\mathcal{S}_m$, $\mathcal{S}_r$ and $\mathcal{S}_\nu$ are respectively the matter, radiation and neutrino actions, α, $\tilde{\alpha}$ and β are constants, and ζ is a constant which does not appear as a model parameter. In the discussion to follow, it will be clear that $\tilde{\alpha}$ controls slow roll such that $\tilde{\alpha} \ll 1$, β is linked to the scale of inflation and α is related to post-inflationary dynamics.[146,147,168]

As demonstrated in Refs. 207 and 208, in the presence of a nonminimal coupling between neutrino matter and the scalar field, the conservation equation for massive neutrinos has the following form (see Sec. 2.3)

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\partial \ln m_\nu}{\partial \phi}(\rho_\nu - 3p_\nu)\dot{\phi}, \tag{122}$$

and for the model under consideration the continuity equation for massive neutrinos is given by[146,168]

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \tilde{\gamma}\alpha(\rho_\nu - 3p_\nu)\frac{\dot{\phi}}{M_{\rm Pl}}. \tag{123}$$

Comparing Eqs. (122) and (123), we find that

$$m_\nu = m_{\nu,0} \mathrm{e}^{\tilde{\gamma}\alpha\phi/M_{\rm Pl}}, \tag{124}$$

where $m_{\nu,0} = m_\nu(\phi = 0)$.

Hence, finally we end up with massive neutrino matter with exponentially growing neutrino masses. This is a phenomenological set up with arrangements such that $m_\nu(z = 0) \sim 1$ eV. In this case, the neutrino matter would be relevant only at late times, where it might take over the field and build the minimum in its potential. In the following discussion we shall transform the field to a canonical form, in order to clearly understand the possibility of slow roll realization at early epochs.

3.3.1. *Canonical form*

Let us now consider the transformation to canonical field σ through

$$\sigma = \Bbbk(\phi), \tag{125}$$

$$k^2(\phi) = \left(\frac{\partial \Bbbk}{\partial \phi}\right)^2, \tag{126}$$

where $k^2(\phi)$ is given by (119). Using (126) one can transform the action (118) to a canonical form as[146]

$$\mathcal{S}_E = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - V(\Bbbk^{-1}(\sigma))\right] + \mathcal{S}_m + \mathcal{S}_r + \mathcal{S}_\nu(\mathcal{C}^2 g_{\alpha\beta};\Psi_\nu). \tag{127}$$

The canonical field σ can be expressed in terms of the noncanonical field ϕ[146] as

$$\frac{\sigma(\phi)}{M_{\rm Pl}} = \frac{\alpha\phi}{\tilde{\alpha}M_{\rm Pl}} - \frac{1}{\tilde{\alpha}}\ln\{2\alpha^2 + \mathrm{e}^{\alpha\phi/M_{\rm Pl}}\beta^2(\alpha^2+\tilde{\alpha}^2) + 2\alpha\sqrt{(1+\mathrm{e}^{\alpha\phi/M_{\rm Pl}}\beta^2)(\alpha^2+\mathrm{e}^{\alpha\phi/M_{\rm Pl}}\beta^2\tilde{\alpha}^2)}\}$$

$$+\frac{1}{\alpha}\ln\{\alpha^2+\tilde{\alpha}[\tilde{\alpha}+2\mathrm{e}^{\alpha\phi/M_{\rm Pl}}\beta^2\tilde{\alpha}$$
$$+2\sqrt{(1+\mathrm{e}^{\alpha\phi/M_{\rm Pl}}\beta^2)(\alpha^2+\mathrm{e}^{\alpha\phi/M_{\rm Pl}}\beta^2\tilde{\alpha}^2)}]\}+C, \tag{128}$$

where C is an integration constant. Choosing $\sigma(\phi=0)=0$ gives

$$C=\frac{1}{\tilde{\alpha}}\ln\{2\alpha^2+\beta^2(\alpha^2+\tilde{\alpha}^2)+2\alpha\sqrt{(1+\beta^2)(\alpha^2+\beta^2\tilde{\alpha}^2)}\}$$
$$-\frac{1}{\alpha}\ln\{\alpha^2+\tilde{\alpha}[\tilde{\alpha}+2\beta^2\tilde{\alpha}+2\sqrt{(1+\beta^2)(\alpha^2+\beta^2\tilde{\alpha}^2)}]\}. \tag{129}$$

Next we shall consider the case $\tilde{\alpha}<1$ and $\alpha\gg\tilde{\alpha}$, for reasons that will become clear shortly. In the small field approximation ($\phi\ll -2M_{\rm Pl}\ln\beta/\alpha$), we find that[146]

$$k^2(\phi)\approx\frac{\alpha^2}{\tilde{\alpha}^2}, \tag{130}$$

which along with (128) allows us to express σ in terms of ϕ, namely

$$\sigma(\phi)\approx\frac{\alpha}{\tilde{\alpha}}\phi. \tag{131}$$

Finally, in the limit under consideration, the potential gets expressed through the canonical field as

$$V_s(\sigma)\approx M_{\rm Pl}^4\mathrm{e}^{-\tilde{\alpha}\sigma/M_{\rm Pl}}, \tag{132}$$

with $\tilde{\alpha}$ as the slope of the potential, which clearly shows that the potential (120) can give rise to slow roll for $\tilde{\alpha}<1$ at early times. The numerical values of the parameter $\tilde{\alpha}$ can be determined by observations as done in the following discussion.

In the large field approximation ($\phi\gg -2M_{\rm Pl}\ln\beta/\alpha$), we have[146]

$$k^2(\phi)\approx 1, \tag{133}$$

which using (128) leads to the expression for the canonical field:

$$\sigma\approx\phi-\frac{2}{\tilde{\alpha}}\ln\left(\frac{\beta}{2}\right)+\frac{2}{\alpha}\ln\left(\frac{\tilde{\alpha}\beta}{\alpha+\tilde{\alpha}}\right). \tag{134}$$

As a result, in the large field limit, the potential reduces to

$$V_l(\sigma)\approx V_{l0}\mathrm{e}^{-\alpha\sigma/M_{\rm Pl}}, \tag{135}$$

$$V_{l0}=M_{\rm Pl}^4\left(\frac{\beta}{2}\right)^{-2\alpha/\tilde{\alpha}}+\left(\frac{\tilde{\alpha}\beta}{\alpha+\tilde{\alpha}}\right)^2. \tag{136}$$

Hence, at late times the potential acquires the scaling form as it should be. In what follows we shall investigate inflation in detail.

3.3.2. *Inflation*

For the scenario under consideration the slow-roll parameters can be easily cast in terms of the noncanonical field ϕ as[146,147,168]

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{1}{V}\frac{{\rm d}V}{{\rm d}\sigma}\right)^2 = \frac{M_{\rm Pl}^2}{2k^2(\phi)}\left(\frac{1}{V}\frac{{\rm d}V}{{\rm d}\phi}\right)^2 = \frac{\alpha^2}{2k^2(\phi)}, \tag{137}$$

$$\eta = \frac{M_{\rm Pl}^2}{V}\frac{{\rm d}^2V}{{\rm d}\sigma^2} = 2\epsilon - \frac{M_{\rm Pl}}{\alpha}\frac{{\rm d}\epsilon(\phi)}{{\rm d}\phi}, \tag{138}$$

$$\xi^2 = \frac{M_{\rm Pl}^4}{V^2}\frac{{\rm d}V}{{\rm d}\sigma}\frac{{\rm d}^3V}{{\rm d}\sigma^3} = 2\epsilon\eta - \frac{\alpha M_{\rm Pl}}{k^2}\frac{{\rm d}\eta}{{\rm d}\phi}. \tag{139}$$

For $\alpha \gg 1$ and $\tilde{\alpha} \ll 1$, the slow-roll parameters become

$$\epsilon = \frac{\tilde{\alpha}^2}{2}(1+X), \quad \eta = \epsilon + \frac{\tilde{\alpha}^2}{2} \quad \text{and} \quad \xi^2 = 2\tilde{\alpha}^2\epsilon, \tag{140}$$

where $X = \beta^2 {\rm e}^{\alpha\phi/M_{\rm Pl}}$.

We can compute the power spectra of curvature and tensor perturbations using the following expressions:

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_*}\right)^{n_{\rm s}-1+(1/2){\rm d}n_{\rm s}/{\rm d}\ln k\ln(k/k_*)}, \tag{141}$$

$$\mathcal{P}_{\rm t}(k) = A_{\rm t}\left(\frac{k}{k_*}\right)^{n_{\rm t}}, \tag{142}$$

where $A_{\rm s}, A_{\rm t}, n_{\rm s}, n_{\rm t}$ and ${\rm d}n_{\rm s}/{\rm d}\ln k$ denote the scalar amplitude, tensor amplitude, scalar spectral index, tensor spectral index and its running respectively. The number of e-foldings in the model is given by[146,147,168]

$$\mathcal{N} \approx \frac{1}{\tilde{\alpha}^2}\left[\ln(1+X^{-1}) - \ln\left(1+\frac{\tilde{\alpha}^2}{2}\right)\right], \tag{143}$$

which for $\tilde{\alpha} \ll 1$ takes the following form:

$$\mathcal{N} \approx \frac{1}{\tilde{\alpha}^2}\ln(1+X^{-1}), \tag{144}$$

which then yields

$$\epsilon(\mathcal{N}) = \frac{\tilde{\alpha}^2}{2}\frac{1}{1-{\rm e}^{-\tilde{\alpha}^2\mathcal{N}}}. \tag{145}$$

Let us note that small field approximation corresponds to the case $\tilde{\alpha}^2 \gg 1/\mathcal{N}$ (or $X \ll 1$), which implies $\epsilon = \eta/2 = \tilde{\alpha}^2/2$. On the other hand, in the large field limit ($X \gg 1$), we have $\epsilon = \eta/2 = \tilde{\alpha}^2 X/2$, in which case $\tilde{\alpha}^2 \ll 1/\mathcal{N}$. The transition between the two limits takes place for $\tilde{\alpha}^2 \approx 1/\mathcal{N}$.

We can then cast the tensor-to-scalar ratio (r), scalar spectral index ($n_{\rm s}$) and the running of spectral index (${\rm d}n_{\rm s}/{\rm d}\ln k$) through $\tilde{\alpha}$ & $\mathcal{N}$ as[147]

$$r(\mathcal{N},\tilde{\alpha}) \approx 16\epsilon(\mathcal{N}) = \frac{8\tilde{\alpha}^2}{1-{\rm e}^{-\tilde{\alpha}^2\mathcal{N}}}, \tag{146}$$

$$n_s(\mathcal{N}, \tilde{\alpha}) \approx 1 - 6\epsilon + 2\eta = 1 - \tilde{\alpha}^2 \coth\left(\frac{\tilde{\alpha}^2\mathcal{N}}{2}\right), \tag{147}$$

$$\frac{dn_s}{d\ln k} \approx 16\epsilon\eta - 24\epsilon^2 - 2\xi^2 = -\frac{\tilde{\alpha}^4}{2\sinh^2\left(\frac{\tilde{\alpha}^2\mathcal{N}}{2}\right)}. \tag{148}$$

In Fig. 5, we present the tensor-to-scalar ratio (r) versus $\tilde{\alpha}$, for a given number of e-foldings $\mathcal{N}$. The shaded region marks the allowed values of r in 1σ confidence level in accordance with the findings of BICEP2[61] collaboration, i.e. $r = 0.2^{+0.07}_{-0.05}$. Thus, from this figure we deduce that the values of r allowed by the BICEP2 can be obtained by tuning the parameter $\tilde{\alpha}$, for example $r \approx 0.2$ if $\tilde{\alpha} = 0.12$ and $\mathcal{N} = 60$. Using expressions (147) and (148) we can then find the corresponding values, namely $n_s = 0.965$ and $dn_s/d\ln k = -0.000522$.

Figure 6 shows the 1σ (blue) and 2σ (cyan) likelihood contours on the $n_s - r$ plane for the observations $Planck + WP + highL + BICEP2$,[61] as well as the 1σ (red) and 2σ (pink) contours from the observations $Planck + WP + highL$.[220] On top, we display the predictions of the model under consideration. For example, the black solid curves bound the region predicted in our model for e-foldings between $\mathcal{N} = 50$ and $\mathcal{N} = 70$ and for the parameter $\tilde{\alpha}$ ranging from 0^+ to 0.175. Figure 6 clearly shows that we can obtain a tensor-to-scalar ratio well within the 1σ (blue) confidence level by choosing the suitable values of the parameter $\tilde{\alpha}$. Moreover, using $r = -8n_t$, we also find the range of n_t as $-0.0338 \le n_t \le -0.0188$ for the given BICEP2[61] range of r in 1σ confidence level.

As for the COBE normalized value of density perturbations, we use the following fitting function[29]:

$$A_s = 1.91 \times \frac{10^{-5} e^{1.01(1-n_s)}}{\sqrt{1 + 0.75r}}. \tag{149}$$

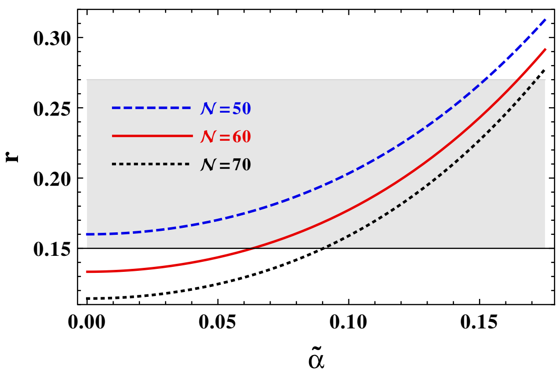


Fig. 5. (Color online) Tensor-to-scalar ratio (r) versus $\tilde{\alpha}$, for different e-foldings $\mathcal{N}$. Blue (dashed), red (solid) and black (dotted) lines correspond to $\mathcal{N} = 50$, 60 and 70, respectively. The shaded region represents the BICEP2 constraint on r at 1σ confidence level, that is $r = 0.2^{+0.07}_{-0.05}$ (see Ref. 61).

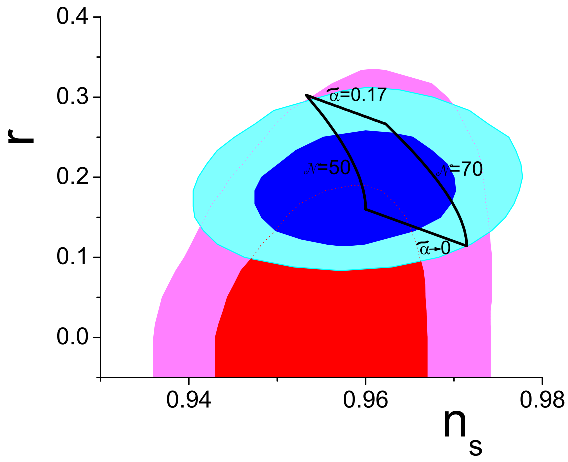


Fig. 6. 1σ (red) and 2σ (pink) contours for *Planck* + *WP* + *highL* data, and 1σ (blue) and 2σ (cyan) contours for *Planck* + *WP* + *highL* + *BICEP*2 data, on the $n_s - r$ plane. The black solid curves mark the region predicted in our model for the parameter $\tilde{\alpha}$ between 0^+ and 0.175, and for e-foldings between $\mathcal{N} = 50$ and $\mathcal{N} = 70$. The upper line ($\tilde{\alpha} = 0.17$) is for $\mathcal{N}$ from 50 to 70, the right curve ($\mathcal{N} = 70$) is for $\tilde{\alpha}$ from 0^+ to 0.17, the lower line ($\tilde{\alpha} \to 0$) is for $\mathcal{N}$ from 50 to 70, and the left curve ($\mathcal{N} = 50$) is for $\tilde{\alpha}$ from 0^+ to 0.17 (see Ref. 147).

According to BICEP2,[61] $r = 0.2^{+0.07}_{-0.05}$ whereas *Planck* 2013 results[220] indicated that $n_s = 0.9603 \pm 0.0073$. Hence, the COBE normalized value of density perturbations for the best fit values of r and n_s taken from the BICEP2[61] and *Planck*[220] observations is given by 1.8539×10^{-5}.

The scalar perturbation spectrum

$$A_s^2(k) = \frac{V}{(150\pi^2 M_{\rm Pl}^4 \epsilon)}, \tag{150}$$

at the horizon crossing ($k = k_* = a_* H_*$) is

$$A_s^2(k_*) = 7^{n_{s*}-1}\delta_H^2. \tag{151}$$

We mention that the energy scale of inflation is directly related to r (with a weak dependence on n_S). It can be represented by the following expression:

$$V_*^{1/4} = \left(\frac{7^{n_{s*}-1} r_*}{1 - 0.07 r_* - 0.512 n_{s*}}\right)^{1/4} 2.75 \times 10^{16}\,\text{GeV}. \tag{152}$$

Using expression (152), for $r = 0.2$ and $n_s = 0.9603$, we find the energy scale of inflation to be 2.157×10^{16} GeV. Additionally, COBE normalization also allows us to obtain a relation between the parameters $\tilde{\alpha}$, β and e-foldings $\mathcal{N}$, namely[147]

$$\frac{\beta^2 \sinh^2\left(\frac{\tilde{\alpha}^2 \mathcal{N}}{2}\right)}{\tilde{\alpha}^2} = 6.36 \times 10^{-8}. \tag{153}$$

As mentioned above, the nucleosynthesis constraint (*Planck* results[107]), puts a bound $\alpha \gtrsim 20$ which with $\tilde{\alpha} \ll 1$ tells us that inflation ends in the region of large values of X. Indeed, in the large-field slow-roll regime, $\epsilon = \eta = \tilde{\alpha}^2 X/2 \Rightarrow X_{\rm end} = 2/\tilde{\alpha}^2 \gg 1$, which leads to $k^2(\phi) \simeq \alpha^2/(\tilde{\alpha}^2 X) \Rightarrow k_{\rm end} \simeq \alpha/\sqrt{2}$.

Let us comment on the small and large field limit. Remembering that the two regions are separated by the boundary $\tilde{\alpha} = \sqrt{1/\mathcal{N}}$, we conclude that if inflation begins in the large field region, $\tilde{\alpha}$ needs to be small in order to get the required number of e-foldings. In case inflation commences around the boundary, the range of slow roll is larger and we might improve upon the numerical values of $\tilde{\alpha}$ for the given number of e-foldings. And this should give rise to larger values of r.

We then turn to the computation of the quantities of interest at the commencement of inflation:

$$X_{\rm in} = \frac{1}{(\mathrm{e}^{\tilde{\alpha}^2\mathcal{N}} - 1)}, \tag{154}$$

which yields the corresponding potential value

$$V_{\rm in} = M_{\rm Pl}^4 \beta^2 (\mathrm{e}^{\tilde{\alpha}^2\mathcal{N}} - 1). \tag{155}$$

Eliminating β in favor of $\tilde{\alpha}$ and $\mathcal{N}$ in Eq. (153) we have

$$V_{\rm in} = \frac{2.5 \times 10^{-7} \tilde{\alpha}^2 M_{\rm Pl}^4}{(1 - \mathrm{e}^{-\tilde{\alpha}^2\mathcal{N}})}. \tag{156}$$

$V_{\rm in}^{1/4}$ provides the scale of inflation and should agree with (152).

It is important to relate the quantities of interest at the end and at the beginning of inflation. We find

$$\frac{X_{\rm in}}{X_{\rm end}} = \frac{V_{\rm end}}{V_{\rm in}} = \frac{\tilde{\alpha}^2}{2(\mathrm{e}^{\tilde{\alpha}^2\mathcal{N}} - 1)}, \tag{157}$$

which in the region of large field reduces to

$$\frac{X_{\rm in}}{X_{\rm end}} = \frac{V_{\rm end}}{V_{\rm in}} = \frac{1}{2\mathcal{N}}. \tag{158}$$

Since during inflation $3H^2 M_{\rm Pl}^2 \approx V$, we also get the ratio $H_{\rm end}/H_{\rm in}$ using (157), which gives the estimation for $H_{\rm end}$ as

$$H_{\rm end} = \frac{M_{\rm Pl}\beta\tilde{\alpha}}{\sqrt{6}} = \frac{1.02 \times 10^{-4} \tilde{\alpha}^2 M_{\rm Pl}}{\sinh\left(\dfrac{\tilde{\alpha}^2\mathcal{N}}{2}\right)}. \tag{159}$$

The above quoted estimates are important for the computation of radiation energy density and its ratio to the field energy density at the end of inflation.

3.3.3. *Relic gravitational wave spectrum*

As shown in Sec. 2.5, the spectral energy density of relic gravitational waves $\tilde{\rho}_{\rm g}(k)$ generated during the transition from de Sitter to post-inflationary phase, crucially depends upon the post-inflationary equation-of-state parameter w[48]:

$$\tilde{\rho}_{\rm g}(k) \propto k^{1-2|c|}, \tag{160}$$

where c is defined in Sec. 2.5.

In the present scenario, inflation essentially follows by the kinetic regime with $w = w_\phi = 1$, which implies a blue spectrum of gravitational wave background $\rho_{\rm g} \propto k$. Since $n_{\rm t} = -r/8$ is small, we ignored it when we assumed inflation to be exactly exponential. Therefore, the blue spectrum in our case is related to the kinetic regime that follows quintessential inflation.

As demonstrated in Sec. 2.5 and in Refs. 37, 38, 45 and 48, the gravitational wave amplitude enhances during the kinetic regime, which might lead to violation of the nucleosynthesis constraint at the commencement of the radiative regime, depending upon the length of the kinetic regime. Using the condition (106) with

$$h^2_{\rm GW} = \frac{H^2_{\rm in}}{8\pi M^2_{\rm Pl}} = \frac{3.315\times 10^{-9}\tilde{\alpha}^2}{1-{\rm e}^{-\tilde{\alpha}^2\mathcal{N}}}, \tag{161}$$

and the nucleosynthesis constraint (107), allows us to estimate the radiation energy density at the end of inflation as

$$\rho_{\rm r,end} \geq \frac{3.517\times 10^{-14} M^4_{\rm Pl}\tilde{\alpha}^6 {\rm e}^{\tilde{\alpha}^2\mathcal{N}/2}}{\sinh^3\left(\dfrac{\tilde{\alpha}^2\mathcal{N}}{2}\right)}. \tag{162}$$

We can also estimate $T_{\rm end} = \rho^{1/4}_{r,\rm end}$ using (162). Now the bound on r from BICEP2[61] gives the bound on $\tilde{\alpha}$ as $0.063 \leq \tilde{\alpha} \leq 1.83$ for $\mathcal{N} = 60$. For $\tilde{\alpha} = 0.12$ and $\mathcal{N} = 60$, $r \approx 0.2$ and we get the bound on the temperature at the end of inflation as $T_{\rm end} \geq 6.65 \times 10^{13}$ GeV. This condition cannot be fulfilled if reheating takes place through gravitational particle production. As shown in Sec. 2.4, instant preheating[169–171] can be implemented[48,146] in this case. Applying the constraint (162) on (85), we can derive limits on the parameter space of the coupling $g \gtrsim 6\alpha \times 10^{-5}$ and $h \gtrsim 2\sqrt{g} \times 10^{-6}$.[146]

Keeping in mind observations such as LIGO and LISA, it is convenient to define the dimensionless spectral energy density parameter

$$\Omega_{\rm GW}(k) = \frac{\tilde{\rho}_{\rm g}(k)}{\rho_{\rm c}}, \tag{163}$$

where $\rho_{\rm c}$ is the critical energy density and (for detailed calculations, we refer the reader to Ref. 48)

$$\Omega^{(\rm MD)}_{\rm GW} = \frac{3}{8\pi^3} h^2_{\rm GW}\Omega_{\rm m0}\left(\frac{\lambda}{\lambda_{\rm h}}\right)^2, \quad \lambda_{\rm MD} < \lambda \leq \lambda_{\rm h}, \tag{164}$$

$$\Omega^{(\rm RD)}_{\rm GW}(\lambda) = \frac{1}{6\pi} h^2_{\rm GW}\Omega_{\rm r0}, \quad \lambda_{\rm RD} < \lambda \leq \lambda_{\rm MD}, \tag{165}$$

$$\Omega^{(\rm kin)}_{\rm GW}(\lambda) = \Omega^{(\rm RD)}_{\rm GW}\left(\frac{\lambda_{\rm RD}}{\lambda}\right), \quad \lambda_{\rm kin} < \lambda \leq \lambda_{\rm RD}, \tag{166}$$

with

$$\lambda_{\rm h} = 2cH_0^{-1}, \tag{167}$$

$$\lambda_{\rm MD} = \frac{2\pi}{3}\lambda_{\rm h}\left(\frac{\Omega_{\rm r0}}{\Omega_{\rm m0}}\right)^{1/2}, \tag{168}$$

$$\lambda_{\rm RD} = 4\lambda_{\rm h}\left(\frac{\Omega_{\rm m0}}{\Omega_{\rm r0}}\right)^{1/2}\frac{T_{\rm MD}}{T_{\rm rh}}, \tag{169}$$

$$\lambda_{\rm kin} = cH_{\rm kin}^{-1}\left(\frac{T_{\rm rh}}{T_0}\right)\left(\frac{H_{\rm kin}}{H_{\rm rh}}\right)^{1/3}, \tag{170}$$

where "MD," "RD" and "kin" denote the matter, radiation and kinetic energy dominated regimes respectively; H_0, $\Omega_{\rm m0}$ and $\Omega_{\rm r0}$ designate Hubble parameter, matter and radiation energy density parameters at the present epoch. Finally, $T_{\rm rh}$ and $H_{\rm rh}$ are respectively the reheating temperature and Hubble parameter at the time of reheating, which takes place very close to the end of inflation as we saw in Sec. 2.4.

In Fig. 7, we present the spectrum of the spectral energy density of relic gravitational waves with wavelength λ, while sensitivity curves of advanced LIGO[221] and LISA[222] are also depicted. Furthermore, in Fig. 8 we depict the spectrum of relic gravitational waves for different numerical values of the tensor-to-scalar ratio r. Next, expressing $h_{\rm GW}$ in terms of the tensor-to-scalar ratio using (146) and (161), gives $h_{\rm GW}^2 = 3.315\times 10^{-9} r/8$. Hence, the square of the amplitude of gravitational waves is directly proportional to r. Since the spectral energy density parameter $\Omega_{\rm GW}$ is proportional to the square of the amplitude, $\Omega_{\rm GW}$ also increases with r, as can also observe in Fig. 8.

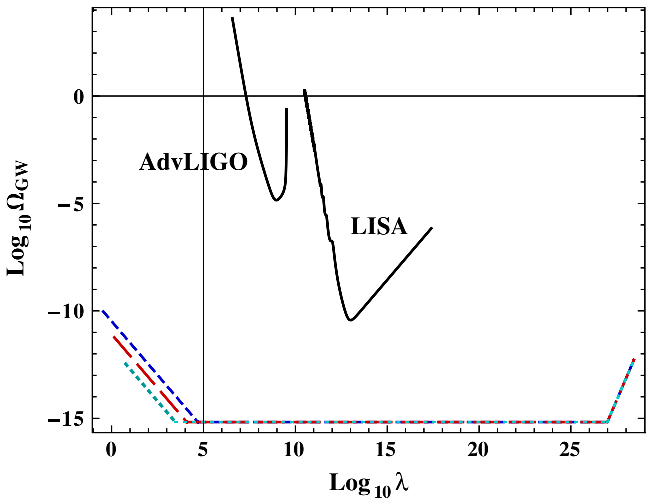


Fig. 7. (Color online) The spectral energy density of the relic gravitational wave background as a function of the wavelength λ. Blue (small dashed), red (long dashed) and cyan (dotted) lines correspond respectively to reheating temperatures 7×10^{13} GeV, 2.5×10^{14} GeV and 8×10^{14} GeV. We have considered $\tilde{\alpha} = 0.12$ and $\mathcal{N} = 60$. Black solid lines represent the sensitivity curves of advanced LIGO and LISA.

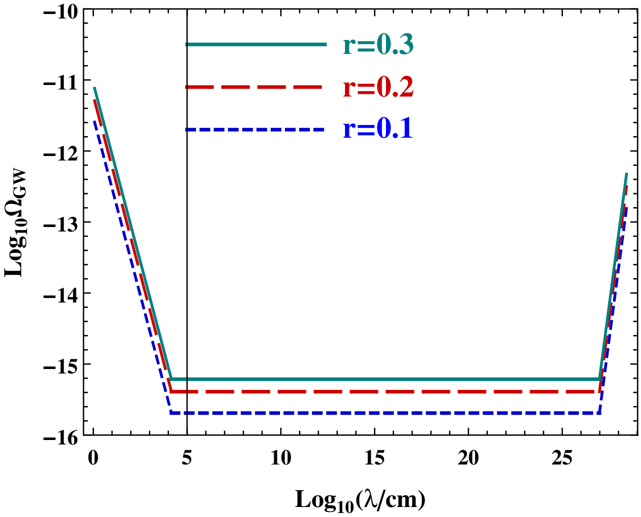


Fig. 8. (Color online) The spectral energy density of the relic gravitational wave background as a function of the wavelength λ. Blue (small dashed), red (long dashed) and cyan (solid) lines respectively correspond to tensor-to-scalar ratio $r = 0.1, 0.2$ and 0.3, with reheating temperature 10^{14} GeV. We have considered $\tilde{\alpha} = 0.12$ and $\mathcal{N} = 60$.

3.3.4. *Evading Lyth bound*

In the preceding discussion we have shown that the scale of inflation depends upon the tensor-to-scalar ratio of perturbations r. It turns out that the range of inflation also crucially depends upon this ratio, giving rise to super Planckian excursion of the field for large values of r, irrespectively of the underlying model of inflation.

Indeed, in case of single canonical scalar field φ model, the number of e-folds is given by

$$\mathcal{N} = \frac{1}{M_{\mathrm{Pl}}^2}\int_{\varphi_{\mathrm{end}}}^{\varphi_{\mathrm{in}}} \frac{V(\varphi)}{V'(\varphi)}\mathrm{d}\varphi \equiv \frac{1}{M_{\mathrm{Pl}}}\int_{\varphi_{\mathrm{end}}}^{\varphi_{\mathrm{in}}} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon_0}}, \tag{171}$$

where ϵ_0 is the standard slow-roll parameter. This expression leads to the following inequality:

$$\mathcal{N} \lesssim \frac{|\varphi_{\mathrm{in}} - \varphi_{\mathrm{end}}|}{M_{\mathrm{Pl}}\sqrt{2\epsilon_{0\mathrm{min}}}}. \tag{172}$$

For simplicity, we assume that slow-roll parameters have monotonous behavior. In that case, $\epsilon_{0\mathrm{min}} \approx \epsilon_{0\mathrm{in}}$, where $\epsilon_{0\mathrm{in}}$ denotes the value of ϵ_0 at the commencement of inflation. Using then the consistency relation $r_\star = 16\epsilon_{0\mathrm{in}}$, with $r_\star$ the tensor-to-scalar ration at the commencement of inflation, and relation (172), gives the bound on the range of inflation known as Lyth bound:

$$\delta\varphi \equiv |\varphi_{\mathrm{in}} - \varphi_{\mathrm{end}}| \gtrsim \mathcal{N} M_{\mathrm{Pl}} \left(\frac{r_\star}{8}\right)^{1/2}, \tag{173}$$

which implies that $\delta\varphi \gtrsim 5M_{\rm Pl}$[65] if $r_\star \gtrsim 0.1$ and $\mathcal{N} = 50$. This super-Planckian field excursion throws a challenge to the framework of effective field theory.

It is important to look for a field theoretic framework which would allow to evade the Lyth bound. Let us show that the bound gets modified in the case of a noncanonical scalar field with the Lagrangian $-1/2k^2(\phi)\partial_\mu\phi\partial^\mu\phi + V$, where $k(\phi)$ is a kinetic function. In this case, the number of e-folds ($\mathcal{N}$) is given by

$$\mathcal{N} = \frac{1}{M_{\rm Pl}^2}\int_{\sigma_{\rm end}}^{\sigma_{\rm in}} \frac{V(\sigma)}{\frac{{\rm d}V(\sigma)}{{\rm d}\sigma}}{\rm d}\sigma = \frac{1}{M_{\rm Pl}^2}\int_{\phi_{\rm end}}^{\phi_{\rm in}} k^2(\phi)\frac{V(\phi)}{V'(\phi)}{\rm d}\phi, \tag{174}$$

which using (137) gives us the bound[66]

$$\mathcal{N} \lesssim \frac{\delta\phi}{M_{\rm Pl}^2}\left|k^2(\phi)\frac{V(\phi)}{V'(\phi)}\right|_{\rm max} = \frac{\delta\phi}{M_{\rm Pl}}\frac{k_{\rm max}}{\sqrt{2\epsilon_{\rm min}}}. \tag{175}$$

Assuming again $r_\star = 16\epsilon_{\rm in}$ and using expression (175), we find the following relation for the range of inflation:

$$\delta\phi \gtrsim \left(\mathcal{N}M_{\rm Pl}\sqrt{\frac{r_\star}{8}}\right)\frac{1}{k_{\rm max}} = \left(\mathcal{N}M_{\rm Pl}\sqrt{\frac{r_\star}{8}}\right)\frac{\tilde{\alpha}}{\alpha}, \tag{176}$$

where we have used the fact that $k_{\rm max} = \alpha/\tilde{\alpha}$. The extra multiplicative factor $\tilde{\alpha}/\alpha \ll 1$ in (176) allows for a large range in sub-Planckian region.

We will now check explicitly that the sub-Planckian range is consistent with observations. Let us consider the following ratio[66,147]

$$\frac{V_{\rm end}}{V_{\rm in}} = \frac{\tilde{\alpha}^2}{2({\rm e}^{\tilde{\alpha}^2\mathcal{N}} - 1)} = \frac{r_\star}{16}{\rm e}^{-\tilde{\alpha}^2\mathcal{N}}, \tag{177}$$

which gives[66]

$$\frac{\alpha}{M_{\rm Pl}}|\phi_{\rm in} - \phi_{\rm end}| = \frac{\alpha\delta\phi}{M_{\rm Pl}} = \left|\ln\left[\frac{\tilde{\alpha}^2}{2\left({\rm e}^{\tilde{\alpha}^2\mathcal{N}} - 1\right)}\right]\right| = \left|\ln\left(\frac{r_\star}{16}\right) - \tilde{\alpha}^2\mathcal{N}\right|. \tag{178}$$

Using (146) we find that $r_\star \approx 0.15$ for $\tilde{\alpha} = 0.06$ and $\mathcal{N} = 60$. Considering these values and using Eq. (178), we arrive at the estimate $\delta\phi/M_{\rm Pl} \approx 5/\alpha$. For $\alpha = 20$, $\delta\phi = 0.25M_{\rm Pl}$ which is the maximum value of $\delta\phi$. The latter is consistent with the bound (176). Indeed, using relation (176) and taking $\mathcal{N} = 60$, $r_\star = 0.15$, $\alpha = 20$ and $\tilde{\alpha} = 0.06$, we obtain the bound $\delta\phi \geq 0.0246\, M_{\rm Pl}$. Moreover, one can check that our conclusion holds for the entire observed range of $\tilde{\alpha}$. Hence, we conclude that the model under consideration can evade the super-Planckian Lyth bound. It is interesting to note that the requirement of viable post-inflationary evolution helps in keeping the range of inflation sub-Planckian.

3.3.5. *Late time dynamics*

As we have already mentioned, the late-time exit from the scaling regime in the model at hand, is caused by the nonminimal coupling of the field to massive neutrino matter. Indeed, varying the action (127) with respect to the metric $g_{\mu\nu}$, we obtain the two Friedmann equations:

$$3H^2 M_{\rm Pl}^2 = \frac{1}{2}\dot{\sigma}^2 + V(\sigma) + \rho_m + \rho_r + \rho_\nu, \tag{179}$$

$$(2\dot{H} + 3H^2)M_{\rm Pl}^2 = -\frac{1}{2}\dot{\sigma}^2 + V(\sigma) - \frac{1}{3}\rho_r - p_\nu, \tag{180}$$

where the neutrino pressure p_ν behaves as radiation during the early times but mimics nonrelativistic matter at late times. Varying the action (127) with respect to the field σ leads to its equation of motion[b]:

$$\ddot{\sigma} + 3H\dot{\sigma} = -\frac{\mathrm{d}V(\sigma)}{\mathrm{d}\sigma} - \frac{\partial \ln m_\nu}{\partial \sigma}(\rho_\nu - 3p_\nu), \tag{181}$$

with[146]

$$\frac{\partial \ln m_\nu}{\partial \sigma} = \frac{\tilde{\gamma}\alpha}{M_{\rm Pl}k(\phi)}. \tag{182}$$

Clearly, during the radiative regime the last term in the r.h.s. of Eq. (181) does not contribute, since during that era neutrinos behave like radiation and its energy–momentum tensor is traceless. However, at late times neutrinos behave as nonrelativistic matter, and the nonminimal coupling between the scalar field and neutrinos builds up and crucially transforms the late-time dynamics. We shall use the following ansatz for $w_\nu(z)$ to mimic the said transition[146]:

$$w_\nu(z) = \frac{p_\nu}{\rho_\nu} = \frac{1}{6}\left\{1 + \tanh\left[\frac{\ln(1+z) - z_{\rm eq}}{z_{\rm dur}}\right]\right\}. \tag{183}$$

In the above expression $z_{\rm eq}$ and $z_{\rm dur}$ determine the time and duration of the transition. Since massive neutrinos should be nonrelativistic in the recent cosmological past, we deduce that we need a large value of $z_{\rm dur}$ such that the transition is smooth. Following Refs. 199, 200 and 212, we set $z_{\rm NR} \in (2-10)$ for $m_\nu \in (0.015 - 2.3)\,\mathrm{eV}$.

Let us define the dark energy density parameter as

$$\Omega_{\rm DE} = \Omega_\sigma + \Omega_\nu, \tag{184}$$

where Ω's are the separate density parameters (for definitions see Appendix B). The equation-of-state parameters of the total matter content of the universe, of the scalar-field sector, and of the dark-energy sector, can be written as

$$w_{\rm eff} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}, \tag{185}$$

[b]Variation of $\mathcal{S}_\nu$ with respect to σ leads to

$$\frac{1}{\sqrt{-g}}\frac{\delta \mathcal{S}_\nu}{\delta\sigma} = \frac{1}{\sqrt{-g}}\frac{\delta \mathcal{S}_\nu}{\delta\phi}\frac{\partial\phi}{\partial\sigma} = \frac{\mathcal{C}_{,\phi}}{\mathcal{C}}\frac{T^{(\nu)}}{k(\phi)} = \frac{\tilde{\gamma}\alpha}{M_{\rm Pl}}\frac{T^{(\nu)}}{k(\phi)}.$$

$$w_\sigma = \frac{p_\sigma}{\rho_\sigma}, \tag{186}$$

$$w_{\mathrm{DE}} = \frac{w_{\mathrm{eff}} - \frac{1}{3}\Omega_r}{\Omega_{\mathrm{DE}}}, \tag{187}$$

where

$$p_\sigma = \frac{1}{2}\dot{\sigma}^2 - V(\sigma). \tag{188}$$

In order to perform a detailed phase space analysis one needs to form an autonomous system, as we show in Appendix B (see also Ref. 146). Here we just mention the only relevant stable fixed point for which

$$\Omega_m = 0, \tag{189}$$

$$\Omega_r = 0, \tag{190}$$

$$\Omega_\nu = \frac{-3 + \alpha^2(1+\tilde{\gamma})}{\alpha^2(1+\tilde{\gamma})^2}, \tag{191}$$

$$\Omega_\sigma = \frac{\tilde{\gamma}}{1+\tilde{\gamma}} + \frac{3}{\alpha^2(1+\tilde{\gamma})^2}, \tag{192}$$

and the equation-of-state parameters are given by

$$w_{\mathrm{eff}} = -\frac{\tilde{\gamma}}{1+\tilde{\gamma}}, \tag{193}$$

$$w_\sigma = -\frac{\alpha^2\tilde{\gamma}(1+\tilde{\gamma})}{3+\alpha^2\tilde{\gamma}(1+\tilde{\gamma})}, \tag{194}$$

$$w_\nu = 0. \tag{195}$$

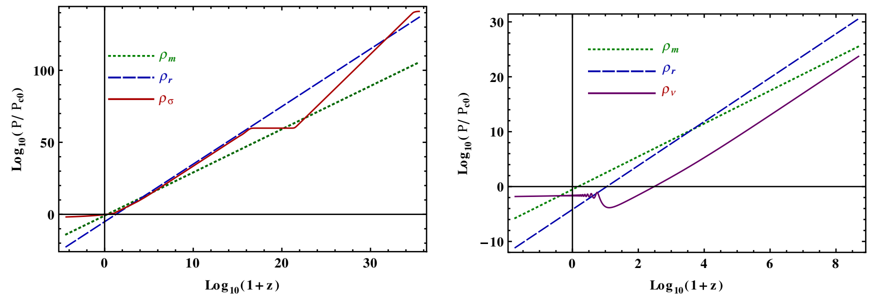
This fixed point is a scaling solution in presence of the coupling, which is accelerating for large $\tilde{\gamma}$ [see Eqs. (193) and (194)]. In the case where the coupling is absent ($\tilde{\gamma} = 0$) one still has a scaling solution, but the corresponding solution is not accelerating.

In Fig. 9(a) and Sec. 3.3.5 we present the post-inflationary evolution of the energy densities of matter (ρ_m), radiation (ρ_{r}), neutrinos (ρ_ν) and scalar field (ρ_σ). As we observe, we have a viable evolution after the inflationary stage.

In Fig. 10(a), we present the universe evolution from the kinetic regime, followed by the radiation, matter and dark energy dominated eras. The sequences are also clear from Fig. 10(b). In Fig. 10(a), we observe that Ω_ν starts growing at the recent past, which is a novel feature introduced by the nonminimal coupling.

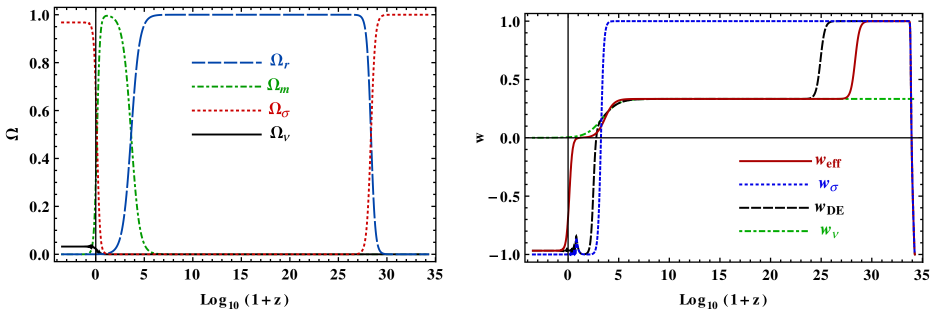
Once again let us emphasize the important role played by massive neutrino matter in our scenario. This late-time interaction of neutrino matter with the scalar field modifies its potential, which in terms of the noncanonical field is given by

$$V_{\mathrm{eff}}(\phi) = V(\phi) + \hat{\rho}_\nu \mathrm{e}^{\tilde{\gamma}\alpha\phi/M_{\mathrm{Pl}}}. \tag{196}$$



(a) (b)

Fig. 9. (Color online) Evolution of various energy densities. ρ_m (green dot-dashed), $\rho_{\rm r}$ (blue dashed), ρ_σ (red solid (upper panel)) and ρ_ν (Purple solid (lower panel)) respectively correspond to matter, radiation, scalar field σ and neutrinos. ρ_{c0} is the present critical energy density of the universe. Figure 9(a) exhibits a tracker behavior of the scalar field, which tracks matter and radiation up to the recent past and then takes over matter and becomes the dominant component of the universe. Section 3.3.5 shows that at late times, when neutrinos become nonrelativistic, ρ_ν takes over radiation and slowly grows thereafter. At the present epoch ρ_ν is still sub-dominant but would take over matter in the future. We have considered $\alpha = 10$, $\tilde{\gamma} = 30$ and $z_{\rm dur} = 10$.



(a) (b)

Fig. 10. (Color online) (a) The evolution of the density parameters of matter (green dot-dashed), radiation (blue long dashed), scalar field σ (red dotted), and neutrinos (black solid). (b) The evolution of the corresponding equation-of-state parameters. We have considered the parameter values $\alpha = 10$, $\tilde{\gamma} = 30$ and $z_{\rm dur} = 3.6$.

This effective potential has a minimum at

$$\phi_{\rm min} = \frac{M_{\rm Pl}}{\alpha(1+\tilde{\gamma})} \ln\left(\frac{M_{\rm Pl}^4}{\tilde{\gamma}\hat{\rho}_\nu}\right), \tag{197}$$

which is the key feature in the scenario under consideration. By setting the model parameters appropriately, it is possible to achieve slow roll of the field around the minimum of the effective potential. Using (197) we obtain the minimum value of the effective potential (196) for $\phi = \phi_{\rm min}$ as

$$V_{\rm eff,min} = (1+\tilde{\gamma})\,\rho_\nu(\phi_{\rm min}), \tag{198}$$

where $\rho_\nu(\phi_{\rm min}) = \hat{\rho}_\nu {\rm e}^{\tilde{\gamma}\alpha\phi_{\rm min}/M_{\rm Pl}}$.

Broadly, since the field has to settle in the minimum of the effective potential during the present epoch, $V_{\rm eff,min} \sim H_0^2 M_{\rm Pl}^2$. Hence, Eq. (198) implies that $\rho_\nu(\phi_{\rm min}) \sim H_0^2 M_{\rm Pl}^2$. It is therefore clear that in the model under consideration the dark energy scale is directly related to the massive neutrino mass scale of recent epoch. We therefore conclude that the scenario at hand leads to successful description of the universe, from inflation to dark energy, in the framework of a single scalar field. However, the stability of the neutrino matter perturbations in the scenario remains to be checked.

4. Summary and Outlook

This review is a pedagogical presentation of the paradigm of quintessential inflation. In Sec. 2, we described the essential concepts required to execute the task of unification of inflation and dark energy. We tried to make clear to the reader that one needs specific features of scalar field dynamics, that would leave intact the bulk of the thermal history of the universe, complementing it at early and late times in a consistent manner. The latter inevitable asks for scaling behavior (after inflation) and exit from it at late times — *a tracker solution*. We have briefly described the realization of the desired features of scalar field dynamics.

Historically, this framework was proposed with the hope to alleviate the fine-tuning problem associated with the cosmological constant. It turns out that a field theoretic set up which includes a fundamental scalar field is plagued with deep issues of theoretical nature *à la* naturalness. In a healthy field theory one expects the *decoupling* of low energy scales from high energy phenomena. For instance, electrodynamics and QCD possess this remarkable property, whereas the standard model of particle physics with the Higgs scalar fails to meet the requirement of naturalness. We have described this important aspect to emphasize that the cosmological constant problem manifests as a problem of naturalness for quintessence field with mass of the order of the present Hubble parameter H_0. In case of a fundamental scalar, naturalness in the high energy regime could be restored by invoking supersymmetry, whereas there is no known way to accomplish the same at low energy.

Since nonminimal coupling plays important role for unification of inflation and dark energy, we have included a subsection on conformal transformation. Leaving technical details to Refs. 172 and 198, we have illustrated the physical equivalence of frames connected with a conformal transformation. Moreover, we included the necessary material needed by dark-energy model building with nonminimal coupling. In models of Type II the post inflationary dynamics is described by steep run-away type potentials. In this case the presence of nonminimal coupling in the Einstein frame triggers a minimum in the potential, whose depth depends upon the coupling α and the slope of the potential λ. By properly adjusting them, it is possible to obtain slow roll around the minimum of the potential. The minimum might occur around the present epoch if we invoke nonminimal coupling with massive neutrino matter. Obviously, this is a phenomenological setting that we have discussed in the review in detail. The latter provides us with a mechanism of exit from the scaling regime, which is valuable for dark energy model building in its own right, irrespectively of quintessential inflation.

Models of quintessential inflation require an alternative reheating mechanism, and instant preheating is specially suitable to this class of scenarios. We have tried to present the estimates of particle production in a model-independent way. Equation (85) is the main result of Sec. 2.4, which is used in Sec. 3 to set the radiation temperature at the end of inflation. We mention here once again that these results can be applied to any other model where the field is nonoscillatory after inflation. The requirement of an efficient reheating mechanism is dictated by the problem posed by relic gravitation waves.

In Sec. 2.5, we also reviewed the basics of quantum generation of gravitational waves during inflation. Their amplitude enhances during the kinetic regime which essentially follows inflation. Inefficient reheating mechanism results into longer kinetic regime, that leads to violation of the nucleosynthesis constraint at the commencement of the radiative regime. While deriving Eq. (107), the main result of this subsection, we omitted many details. This equation, along with (85), fixes the reheating temperature consistently with nucleosynthesis requirements.

In Sec. 3, we first described braneworld quintessential inflation. Unfortunately, this model is ruled out by observations, as the tensor-to-scalar ratio of perturbations in this case is too large, though post-inflationary evolution is satisfactory. To the best of our knowledge, no other mechanism is known at present to implement inflation in models of Type I. We finally discussed Type II models, with a noncanonical kinetic term in the Lagrangian of the scalar field ϕ. The Lagrangian has three parameters, namely $\tilde{\alpha}, \alpha$ and β. In terms of a canonical field $\sigma(\phi)$ for ϕ close to the origin, $V \sim e^{-\tilde{\alpha}\sigma/M_{\rm Pl}}$ which obviously facilitates slow roll for small $\tilde{\alpha}$. Inflation in this model ends for large ϕ such that the field potential has scaling form thereafter, namely $V \sim e^{-\alpha/M_{\rm Pl}}$ where α is fixed using nucleosynthesis constraints ($\alpha \gtrsim 20$). The third parameter β is fixed by COBE normalization. Small (large) field approximation in this case corresponds to $\tilde{\alpha} \gg 1/\mathcal{N}(\tilde{\alpha} \ll 1/\mathcal{N})$. Since r is a monotonously increasing function of $\tilde{\alpha}$, we can reconcile with observations (Planck/BICEP2) depending upon the region where inflation commences. It is interesting to note that the Lyth bound can be evaded in this case provided that $\alpha \gtrsim 24$.

Last but not least, we discussed issues related to relic gravitational waves. The blue spectrum of these waves is generated during the transition from inflation to kinetic regime. This is a generic feature of the scenario at hand, which can be used to falsify the paradigm of quintessential inflation. We hope that future LISA and Adv LIGO would help to settle this issue.

Acknowledgments

We thank R. Adhikari, S. Ahmad, N. Dadhich, S. Das, J. P. Derendinger, C. Q. Geng, E. Guendelman, S. Jhingan, R. Kaul, T. Padmanabhan, S. Panda, V. Sahni, L. Sebastiani, A. A. Sen and V. Soni for useful comments and discussions. M. W. H. acknowledges the local hospitality given by the Theory Group, Department of Physics and Astronomy, University of Lethbridge, Lethbridge, Canada where part of the work was done. M.W.H. also acknowledges CSIR, Govt. of India for financial support through SRF scheme (File No: 09/466(0128)/2010-EMR-I). M.S. thanks the Eurasian International Center for Theoretical Physics, Astana for hospitality where part of the work was accomplished.

Appendix A. Variable Gravity in Jordan Frame

Let us consider the following action with a noncanonical scalar field χ[146,168]

$$\begin{aligned}\mathcal{S}_J = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{F}(\chi)\tilde{R} - \frac{1}{2}\tilde{K}(\chi)\partial^\mu\chi\partial_\mu\chi - \tilde{V}(\chi)\right] \\ + \tilde{\mathcal{S}}_m + \tilde{\mathcal{S}}_r + \tilde{\mathcal{S}}_\nu,\end{aligned} \tag{A.1}$$

with

$$\begin{aligned}\tilde{F}(\chi) &= \chi^2, \\ \tilde{K}(\chi) &= \frac{4}{\tilde{\alpha}^2}\frac{m^2}{\chi^2+m^2} + \frac{4}{\alpha^2}\frac{\chi^2}{\chi^2+m^2} - 6, \\ \tilde{V}(\chi) &= \mu^2\chi^2,\end{aligned}$$

and

$$\begin{aligned}\tilde{\mathcal{S}}_m &= \tilde{\mathcal{S}}_m\left(\frac{\chi^2}{M_{\rm Pl}^2}\tilde{g}_{\alpha\beta};\Psi_m\right), \\ \tilde{\mathcal{S}}_r &= \tilde{\mathcal{S}}_r\left(\frac{\chi^2}{M_{\rm Pl}^2}\tilde{g}_{\alpha\beta};\Psi_r\right), \\ \tilde{\mathcal{S}}_\nu &= \tilde{\mathcal{S}}_\nu\left(\left(\frac{\chi}{M_{\rm Pl}}\right)^{4\tilde{\gamma}+2}\tilde{g}_{\alpha\beta};\Psi_\nu\right),\end{aligned}$$

where tildes denote the quantities in the Jordan frame. As discussed earlier, it proves convenient to work in the Einstein frame. In order to transfer the action (A.1) to Einstein frame, let us consider the following conformal transformation:

$$g_{\mu\nu} = A^{-2}\tilde{g}_{\mu\nu}, \tag{A.2}$$

where $A^{-2} = \tilde{F}(\chi)/M_{\rm Pl}^2$ is the conformal factor and $g_{\mu\nu}$ is the Einstein-frame metric.

Under conformal transformation (A.2) and Eq. (45), the Ricci scalar transforms as

$$\tilde{R} = \frac{\tilde{F}}{M_{\rm Pl}^2}\left\{R + 3\Box\ln\left(\frac{\tilde{F}}{M_{\rm Pl}^2}\right) - \frac{3}{2\tilde{F}^2}g^{\mu\nu}\times\partial_\mu\tilde{F}\partial_\nu\tilde{F}\right\}, \tag{A.3}$$

and the Jordan-frame action (A.1) becomes

$$\mathcal{S}_E = \int \mathrm{d}^4x\sqrt{-g}\left[M_{\mathrm{Pl}}^2\left(\frac{1}{2}R - \frac{1}{2\chi^2}K(\chi)\partial^\mu\chi\partial_\mu\chi\right) - V(\chi)\right] + \mathcal{S}_m + \mathcal{S}_r + \mathcal{S}_\nu\left(\left(\frac{\chi}{M_{\mathrm{Pl}}}\right)^{4\tilde{\gamma}} g_{\alpha\beta};\Psi_\nu\right), \tag{A.4}$$

where

$$V(\chi) = \frac{M_{\mathrm{Pl}}^4\tilde{V}}{\tilde{F}^2}, \tag{A.5}$$

$$K(\chi) = \chi^2\left[\frac{\tilde{K}}{\tilde{F}} + \frac{3}{2}\left(\frac{\partial\ln\tilde{F}}{\partial\chi}\right)^2\right]. \tag{A.6}$$

Finally, for convenience let us define a new noncanonical scalar field ϕ through

$$\chi = \mu\mathrm{e}^{\frac{\alpha\phi}{2M_{\mathrm{Pl}}}}. \tag{A.7}$$

In this case the action (A.4) takes the form of Eq. (118), with

$$\zeta = \left(\frac{\mu}{M_{\mathrm{Pl}}}\right)^{4\tilde{\gamma}}. \tag{A.8}$$

Appendix B. Autonomous System for Variable Gravity Framework

The dimensionless density parameters for matter, radiation, neutrinos and scalar field, are respectively defined as

$$\Omega_m = \frac{\rho_m}{3H^2M_{\mathrm{Pl}}^2}, \tag{B.1}$$

$$\Omega_r = \frac{\rho_{\mathrm{r}}}{3H^2M_{\mathrm{Pl}}^2}, \tag{B.2}$$

$$\Omega_\nu = \frac{\rho_\nu}{3H^2M_{\mathrm{Pl}}^2}, \tag{B.3}$$

$$\Omega_\sigma = \frac{\rho_\sigma}{3H^2M_{\mathrm{Pl}}^2}, \tag{B.4}$$

where

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma). \tag{B.5}$$

In order to examine the cosmological dynamics let us define the following dimensionless variables:

$$x = \frac{\dot{\sigma}}{\sqrt{6}HM_{\mathrm{Pl}}}, \tag{B.6}$$

$$y = \frac{\sqrt{V}}{\sqrt{3}HM_{\mathrm{Pl}}}, \tag{B.7}$$

$$\lambda = -\frac{M_{\rm Pl}}{V(\sigma)}\frac{dV(\sigma)}{d\sigma} = -\frac{M_{\rm Pl}}{k(\phi)}\frac{1}{V(\phi)}\frac{\partial V(\phi)}{\partial\phi} = \frac{\alpha}{k(\phi)}. \tag{B.8}$$

In order to simplify our analysis, we shall use approximations valid at late times. Since in this section we are dealing with late-time cosmology, we can use the late-time approximation of $k(\phi)$. Expanding (119) and keeping up to first order in $\mathrm{e}^{-\alpha\phi/M_{\rm Pl}}$, we find that

$$k^2(\phi) \approx 1 + \frac{\alpha^2-\tilde{\alpha}^2}{\tilde{\alpha}^2\mu_m^2}\mathrm{e}^{-\alpha\phi/M_{\rm Pl}}, \tag{B.9}$$

which indeed satisfies the discussed requirements that after the inflation end $k^2(\phi)$ must go rapidly towards 1 for $\alpha > \tilde{\alpha}$ and $\tilde{\alpha} \ll 1$. Therefore, the variable λ from (B.8) becomes

$$\lambda = \alpha\left[1 + \frac{\alpha^2-\tilde{\alpha}^2}{\tilde{\alpha}^2\mu_m^2}\mathrm{e}^{-\alpha\phi/M_{\rm Pl}}\right]^{-1/2}. \tag{B.10}$$

In summary, using the six dimensionless variables x, y, λ, Ω_m, Ω_r and w_ν, we can transform the cosmological system of equations (179)–(183) into its autonomous form[146]:

$$\frac{\mathrm{d}x}{\mathrm{d}N} = \frac{x}{2}(3w_\nu\Omega_\nu + \Omega_r - 3y^2 - 3) + \frac{3x^3}{2} + \sqrt{\frac{3}{2}}y^2\lambda + \sqrt{\frac{3}{2}}(3w_\nu - 1)\tilde{\gamma}\lambda\Omega_\nu, \tag{B.11}$$

$$\frac{\mathrm{d}y}{\mathrm{d}N} = \frac{y}{2}(3x^2 - \sqrt{6}x\lambda + 3 + 3w_\nu\Omega_\nu + \Omega_r) - \frac{3y^3}{2}, \tag{B.12}$$

$$\frac{\mathrm{d}\Omega_r}{\mathrm{d}N} = -\Omega_r(1 - 3x^2 + 3y^2 - 3w_\nu\Omega_\nu - \Omega_r), \tag{B.13}$$

$$\frac{\mathrm{d}\Omega_m}{\mathrm{d}N} = \Omega_m(3x^2 - 3y^2 + 3w_\nu\Omega_\nu + \Omega_r), \tag{B.14}$$

$$\frac{\mathrm{d}w_\nu}{\mathrm{d}N} = \frac{2w_\nu}{z_{\rm dur}}(3w_\nu - 1), \tag{B.15}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}N} = \sqrt{\frac{3}{2}}x\lambda^2\left(1 - \frac{\lambda^2}{\alpha^2}\right), \tag{B.16}$$

where $N = \ln a$.

The equation-of-state parameters defined in (185)–(187) can be written as

$$w_{\rm eff} = x^2 - y^2 + w_\nu\Omega_\nu + \frac{\Omega_r}{3}, \tag{B.17}$$

$$w_\sigma = \frac{x^2-y^2}{x^2+y^2}, \tag{B.18}$$

$$w_{\rm DE} = \frac{w_{\rm eff} - \frac{1}{3}\Omega_r}{\Omega_{\rm DE}} = \frac{x^2 - y^2 + w_\nu\Omega_\nu}{1 - \Omega_m - \Omega_r}. \tag{B.19}$$

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