Dark Energy: Problems and Outlook

International Workshop on Dark Energy
Jamia Millia Islamia, New Delhi

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- “Big Bang” cosmology: Universe is expanding (recession of galaxies)

- Possible in the framework of General Relativity:

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

- Redshift:

$$1 + z = \frac{a_0}{a} = \frac{\lambda_0}{\lambda}$$

- Spectacularly confirmed: Cosmic Microwave Background (CMB), Primordial Nucleosynthesis

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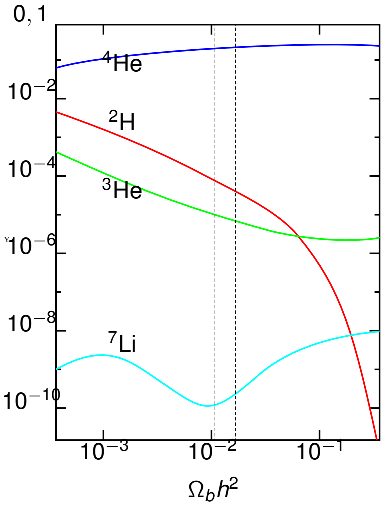
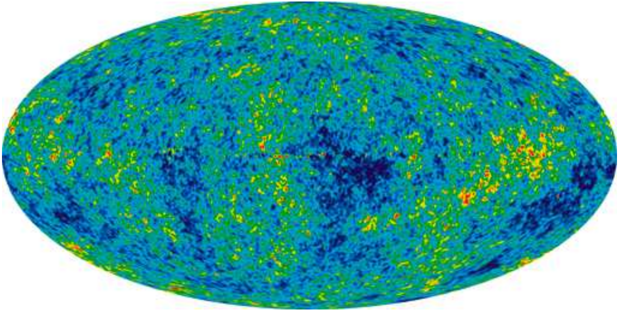
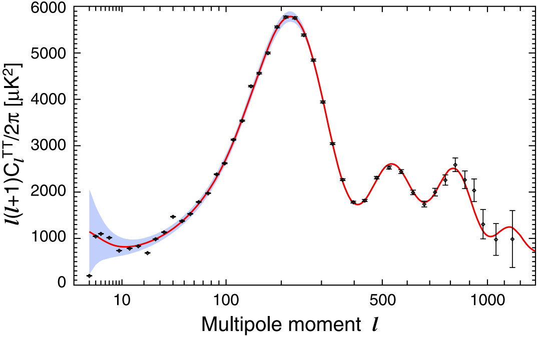


0, 1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
^{4}He
^{2}H
^{3}He
^{7}Li
10^{-3}
10^{-2}
10^{-1}
$\Omega_b h^2$




6000
5000
4000
3000
2000
1000
0
10
100
500
1000
$l(l+1)C_l^{TT}/2\pi$ [μK²]
Multipole moment l

- In "standard" Big Bang cosmology: decelerated expansion

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\,\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- $p_m = 0, \quad p_r = \frac{1}{3} > 0 \quad \Rightarrow \quad \ddot{a} < 0$

- $\frac{\dot{a}}{a} \equiv H \qquad H_0 = 74 \text{ km s}^{-1} \text{ Mpc}^{-1}$

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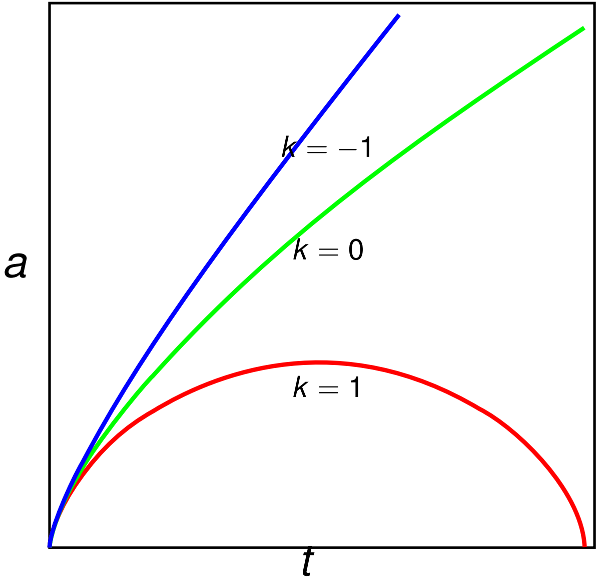
$k = -1$
$k = 0$
$k = 1$
a
t

- Dark Energy paradigm comes from observations: SNIa Luminosity-distances

$$\mathcal{F} = \frac{L}{4\pi d_L^2} \qquad\qquad m - M = 5 \log d_L + 25$$

$$d_L(z) = c\,(1+z)\,H_0^{-1}\,|\Omega_{k,0}|^{-\frac{1}{2}}\,\mathcal{S}\left(|\Omega_{k,0}|^{\frac{1}{2}}\int_0^z \frac{dz'}{h(z')}\right)$$

- Expansion no longer as in standard cosmology

$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \qquad z \sim 0.5$$

- Whet is the origin of this accelerated expansion ?

- We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

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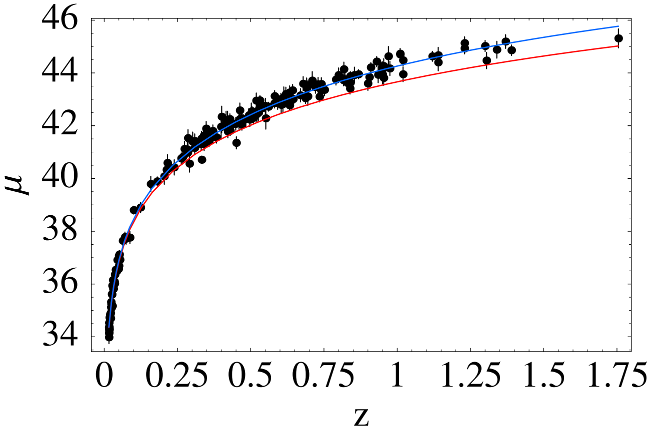

$\Lambda CDM \quad \Omega_{m,0} = 0.3 \quad \Omega_{k,0} = 0$

EdS

- Introduce Dark Energy (DE)

$$
\begin{aligned}
3H^2 &= 8\pi G\,(\rho_m + \rho_{DE}) \\
-2\dot{H} &= 8\pi G\,(\rho_m + \rho_{DE} + p_{DE})
\end{aligned}
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- $h^2(z) = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{m,0}\,(1+z)^3 + \Omega_{DE,0}\; f(z)$

$$f(z) = \exp\left[3\int_0^z dz'\;\frac{1 + w_{DE}(z')}{1+z'}\right]$$

- $w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} < -\frac{1}{3}\,\Omega_{DE}^{-1}$ accelerated expansion

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- Cosmological constant Λ: remarkable simplicity!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$
- There are also observational problems
- $w_\Lambda = -1$, and ρ_Λ is exactly constant
- Other models have generically: $w_{DE}(z)$!!

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- Quintessence: (minimally coupled) scalar field $\phi(t)$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}$$

$$-1 \le w_\phi \le 1 \Leftrightarrow \rho_\phi + p_\phi \ge 0$$

No “phantom”!

- $L = \frac{1}{16\pi G_*}\Big(F(\Phi)\,R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\Big) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad\qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad\qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad\qquad \omega_{BD,0} > 4\times 10^4$$

- $$V = -G_{\text{eff}}\,\frac{M_1\ M_2}{r} \qquad\qquad \text{massless}\ \ \Phi\ \ \text{field}$$

$$G_{\text{eff}} = G_N\left(1 + \frac{1}{2\omega_{BD}+3}\right) \qquad\qquad G_N = \frac{G_*}{F}$$

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Growth of matter perturbations is modified:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\, \rho_m\, \delta_m = 0$$

$$h^2\, \delta''_m + \left(\frac{(h^2)'}{2} - \frac{h^2}{1+z} \right) \delta'_m = \frac{3}{2}(1+z)\frac{G_{\rm eff}}{G}\, \Omega_{m,0}\, \delta_m$$

Perturbations $\delta_m(z)$ must be consistent with background expansion!

Some “DE” clustering

- Modifying gravity ?

Example : $R \to f(R)$

$$f(R) = R - \lambda R_c \left(1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right)$$

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- $A^2 = e^{2\beta\phi/M_{PL}}$ $\qquad V = M^4 \, e^{\left(\frac{M}{\phi}\right)^n}$

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- $G_{\rm eff}(z, k) \Leftrightarrow V(r) = -G_* \frac{M_1 M_2}{r} \left(1 + 2\,\beta^2 \, e^{-m_\phi r}\right)$

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- Matter perturbations can be characterized by the "growth function" $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dx}$

$$\frac{df}{dx} + f^2 + \frac{1}{2}(1 - 3\, w_{\text{eff}})\, f = \frac{3}{2}\frac{G_{\text{eff}}}{G}\Omega_m$$

- A convenient "parameterization" $f = \Omega_m^{\gamma}$. Actually

$$\delta_m(z, k) \Leftrightarrow \gamma = \gamma(z, k)$$

- In ΛCDM: $\gamma \simeq 0.55$
It can be very different in modified gravity models!

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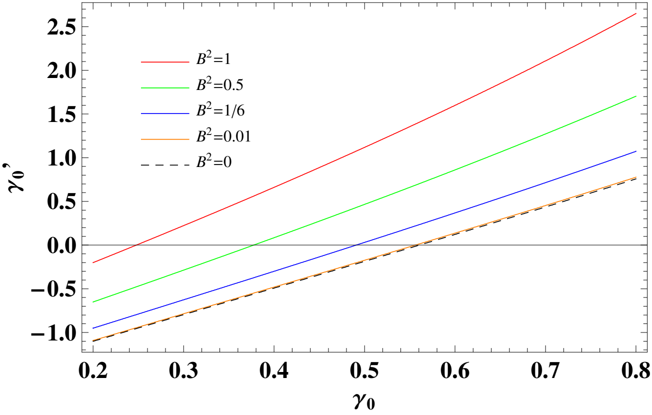
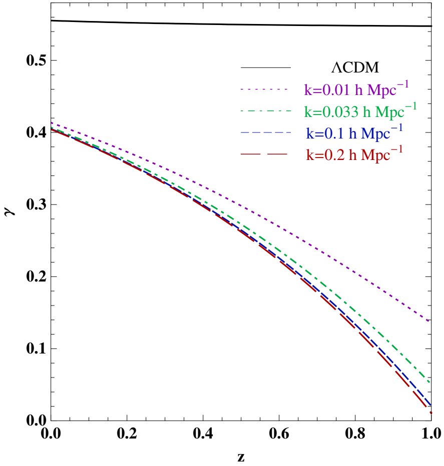
$$C\left(\gamma_0(k),\ \gamma_0'(k),\ \Omega_{m,0},\ w_{\mathrm{eff},0},\ \frac{G_{\mathrm{eff},0}(k)}{G} \equiv 1 + 2B^2\right) = 0$$



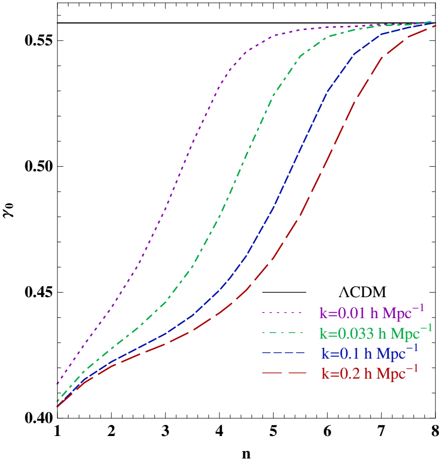
$$\Omega_{m,0} = 0.29 \qquad w_{DE,0} = -1$$

$$f(R) = R - \lambda R_c \frac{x^{2n}}{x^{2n}+1} \qquad x \equiv \frac{R}{R_c}$$



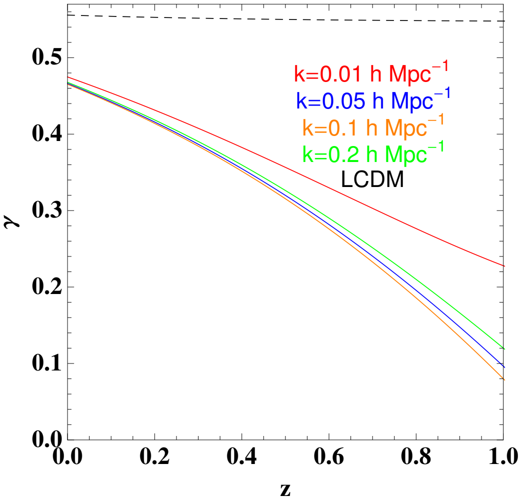
$$n = 1, \quad \lambda = 1.55$$

$$f(R) = R - \lambda R_c \frac{x^{2n}}{(x^{2n}+1)} \qquad x \equiv \frac{R}{R_c}$$

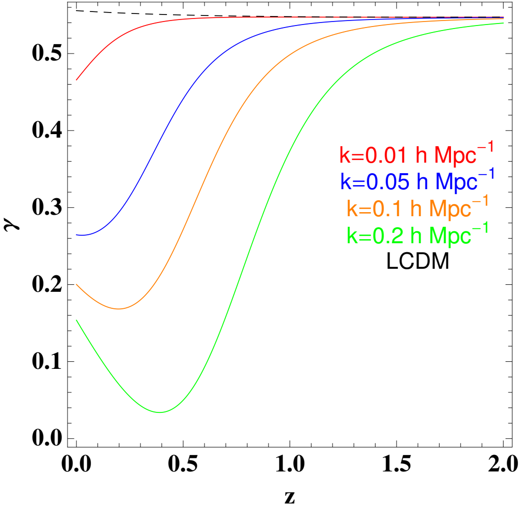


$$\lambda = 1.55$$

$$V(\phi) = M^4[1 - \mu(1 - e^{-\phi/M_{\rm pl}})^n]$$

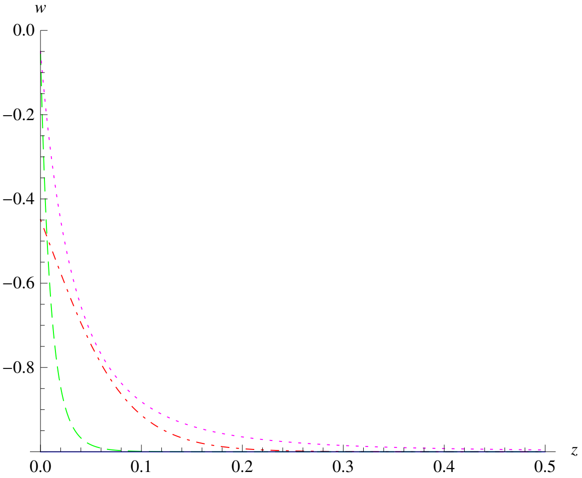


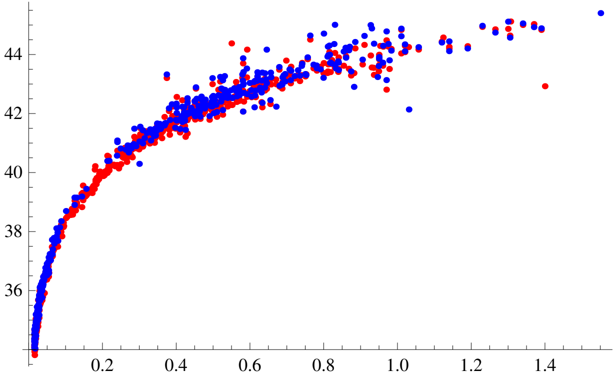
$$n = 0.8 \quad \mu = 0.5 \quad \beta = 1/\sqrt{6}$$



$$n = 0.7 \quad \mu = 0.05 \quad \beta = 1$$

- Universe future is undetermined...forever?




44
42
40
38
36
0.2
0.4
0.6
0.8
1.0
1.2
1.4

- We need complementary probes:

  Supernovae

  Clusters

  Weak lensing

  Baryon Acoustic Oscillations

  Cosmic Microwave Background

  Gamma Ray Bursts?

  Gravitational waves?

  Redshift drift?

- ...theoretical tools: parametrizations, Fisher matrices, Bayesian approach,..

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The art of inducing accelerated expansion

Smooth component in GR

Λ? $w(z)$? $w < -1$?

Quintessence? Chaplygin? Tachyons?... Clustering?

Modifying gravity ?

Scalar-tensor? $f(R)$? Galileon?

Modified growth of perturbations

Challenging cosmological framework ?

What is the universe future ?

Observations will single out viable models

Deep problems induce exciting solutions and challenging questions!